Abstract Expressionism for Parallel Performance

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Abstract

Optimizing Functional Array Language (FAL) compilers for languages such as APL (APEX) and SAC (sac2c), now produce code that outperforms hand-optimized C in both serial and parallel arenas, while retaining the abstract expressionist nature of well-written FAL code.
In this talk, we demonstrate how FAL can now outperform C, in both serial and OpenMP variants, by up to a third, with *no* source code modifications. We also show that modern optimizers can sometimes generate identical loops from substantially different FAL source code.

Talk Layout

- Serial performance: physics relaxation benchmark

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- Parallel performance: physics relaxation benchmark

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- Serial performance: physics relaxation benchmark
- Parallel performance: physics relaxation benchmark
- Wild applause

A Physics Benchmark: Vector Relaxation

- Inputs: temperatures (fixed) at each end of N-element rod

Dyalog APL/S-64 Version 14.1.25324
8-core AMD FX-8350 (Piledriver) @ 4013MHz, 32GB DRAM
Ubuntu 14.04LTS, sac2c Build #18605, gcc 4.8.2-19ubuntu1
100000 iterations of relaxation kernel
100001-element vector argument, N

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Serial Relaxation Timings in Dyalog APL

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- Timings:

APL TD	82.6s
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Serial Relaxation in C Using IF/THEN/ELSE

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for( j=0; j<N; j++) {
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Serial Relaxation in C Using Conditional Expressions

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Serial Relaxation in SAC Using Conditional Expressions

```
res = with {
        ([0] <= [j] < [N]) :
          (0==j)      ? v[j] :
          ((N-1)==j) ? v[j] :
            (v[j-1] + v[j+1])/2.0;
      } : modarray( v);
```

- Timings:

APL TD	82.6s
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C COND	16.4s
SAC COND	12.0s

Serial Relaxation in SAC, Hand-Optimized

Can SAC do better?
Three data-parallel With-Loop partitions:

```
res = with {
      ([0]   <= [j] < [1]) : v[j];
      ([1]   <= [j] < [N-1]) :
         (v[j-1] + v[j+1])/2.0;
      ([N-1] <= [j] < [N]) : v[j];
    } : modarray( v);
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mid = (drop([2],v)+drop([-2],v))/2.0;
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- *Identical* inner loops for `APEX TD` and `SAC HAND`

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- Rotate algorithm in APEX, generated SAC code

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- *ALL* inner loops are identical!

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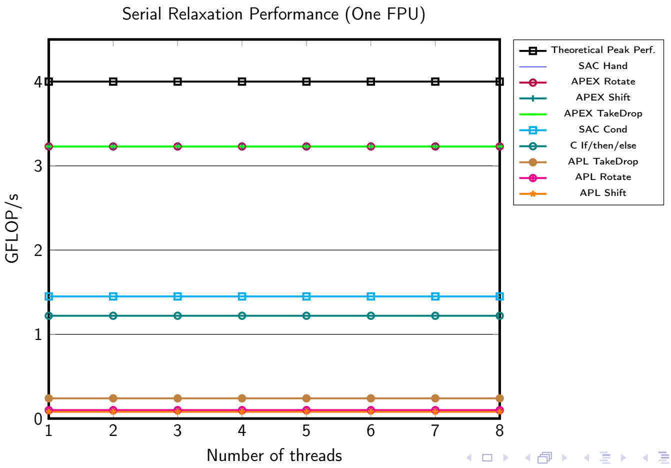
- See paper for stdlib code, here: `http://www.snakeisland.com/abstractexpressionism.pdf`

Serial Performance GFLOPS

- Hard to do better? SAC/APEX approach maximum GFLOPS rate



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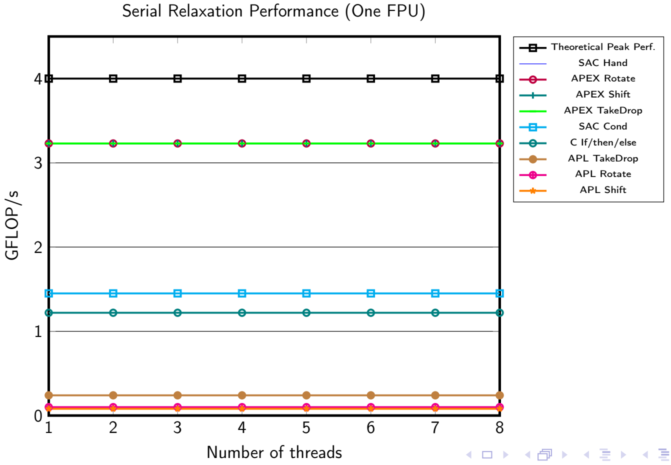
- Let's look at parallel execution



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  after *SOME* `for` statements.

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- Compile with `-fopenmp`

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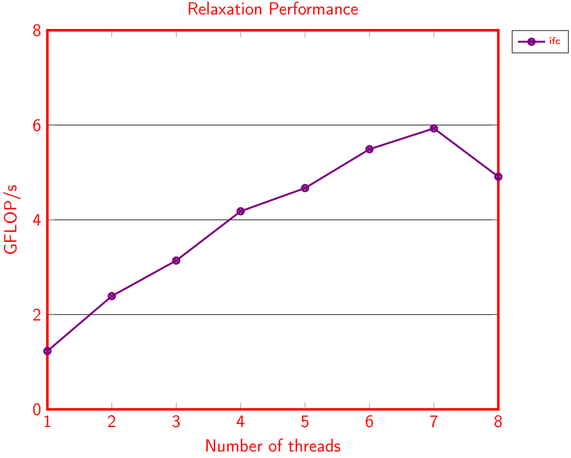

Optimized Parallel Relaxation in C

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- Rationale: Eliminate multiple indexed assigns into "`res`"
- This should improve instruction cache use

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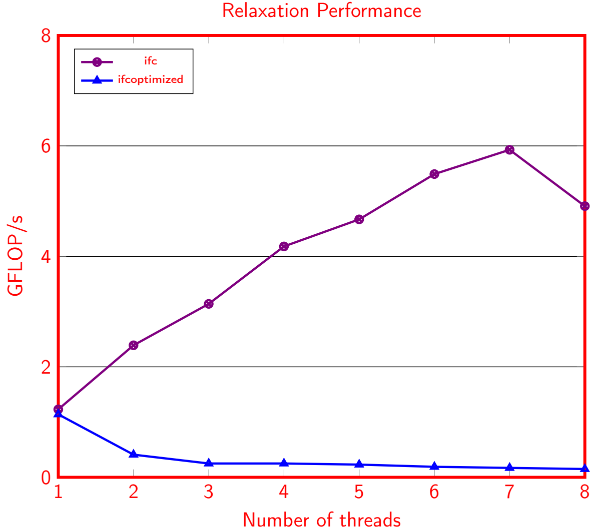

Parallel Relaxation Slowdown in C Post-mortem

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- Bottom line: Writing parallel C code is **NOT** trivial

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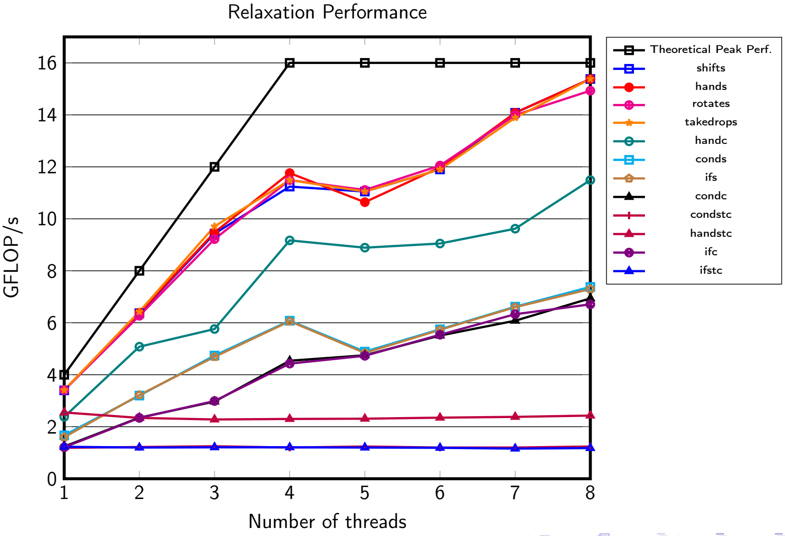
- Abstract expressionist APL matches best SAC code
- SAC and APL beat C by 2.75X in serial environment
- SAC and APL beat Open MP C by 1/3 in parallel environment
- *NO* changes to APL code for parallel execution, unlike C

Serial and Parallel Relaxation Performance

Higher is better



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- and others…

SAC Keys to High-Performance FAL Compilation

- Provide purely functional Intermediate Language (IL)
- Preserve array semantics in IL
- Scalarize small arrays, *e.g.*:
- in Gaussian Elimination pivot, replacing:

```
mat[rowa,rowb;]←mat[rowb,rowa;]
```

by

```
trow←mat[rowa;] ⋄ mat[rowa;]←mat[rowb;] ⋄
mat[rowb;]←trow
```

- ...gives 2X speedup!
- Do scalarization in the compiler, *NOT* in app source code.
- Use array-based optimizations, *e.g.*, with-loop folding (WLF)
- and others...
- Stay tuned for the book!

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Thank you! Questions?