

DYALOG



APL Germany

Language Enhancements

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DYALOG



APL Germany

Language Enhancements

Indexing with nested vectors in APL

Asked 2 years, 3 months ago Modified 2 years, 2 months ago Viewed 132 times



Ask Question



3

I have a vector of vectors that contain some indices, and a character vector which I want to use them on.

```
A←(1 2 3)(3 2 1)
B←'ABC'
```

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I have a vector of vectors that contain some

3

```
A ← (1 2 3)(3 2 1)
B ← 'ABC'
```

I have tried:

```
B[A]
RANK ERROR
B[A]
^
```



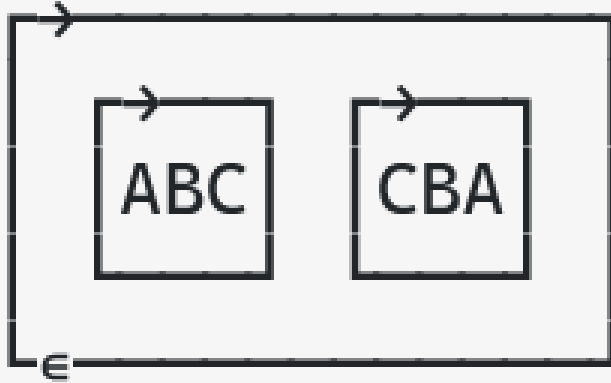
A⊆B
LENGTH ERROR
A⊆B
^

and

A⊆B
LENGTH ERROR
A⊆"B
^



I would like



to be returned, but if i need to find another way,

indexing

apl

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Indexing with Nested Vectors

3 Answers:

✧ $(c^{\cdot\cdot} A) \square^{\cdot\cdot} c B$

✧ $\{B[\omega]\}^{\cdot\cdot\cdot} A$

✧ *Don't do that!*

Possibilities:

✧ $A \square^{\cdot\cdot\cdot} c c B$

✧ $\square \circ B^{\cdot\cdot} c^{\cdot\cdot} A$

✧ $A \square \sim \circ c \sim^{\cdot\cdot} c B$



Indexing with Nested Vectors

3 Answers:

✧ $(c \cdot A) \cdot c \in B$

✧ $\{B[\omega]\} \cdot A$

✧ *Don't do that!*

Possibilities:

✧ $A \cdot c \in B$

✧ $\cdot \circ B \cdot A$

✧ $A(c \cdot \neg \cdot) \cdot c \in B$



Language Enhancements



Core Language Enhancements

Data Transformation

$X \times Y$

ϕY

$X \sqcap Y$

Function Application

$f \neq$

$f \star g$

$f \ddot{o} k$

Function Composition

$f \ddot{o} g$

$f \ddot{o} g$

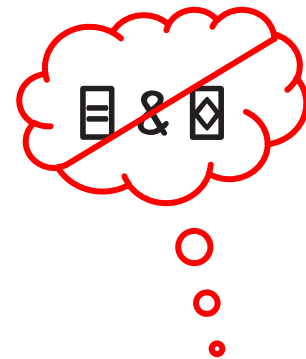
$f \circ g$

Axis Manipulation

$, Y$

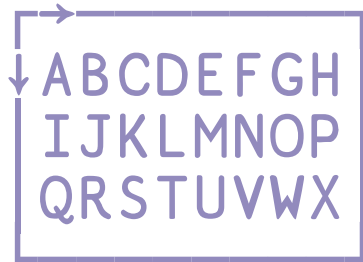
$\bar{,} Y$

$X \phi Y$



Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



$t \leftarrow 3 \quad 8p \square A$

$t[2;4]$

L

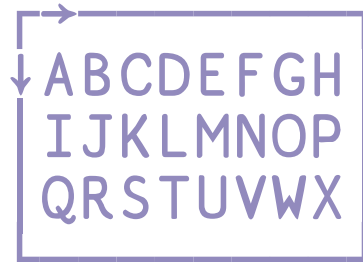
$2 \quad 4 \square t$

L



Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



$t \leftarrow 3 \quad 8p \square A$

$t[2 \quad 1; 4 \quad 1 \quad 7]$

L IO
DAG

$(2 \quad 1)(4 \quad 1 \quad 7) \square t$

L IO
DAG



Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



$p \leftarrow 8p \square A$

$p[2]$

B

$2 \square p$

B



Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



$p \leftarrow 8p \square A$

$p[2 \ 1 \ 7]$

BAG

$2 \ 1 \ 7 \ ?p$

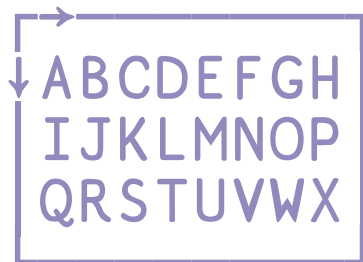
BAG



Data Transformation

Simple Indexing

- Choose Indexing
- Reach Indexing



$t \leftarrow 3 \text{ } 8 \rho \square A$

$t[\text{c}1 \text{ } 8]$

H

$1 \text{ } 8 \square t$

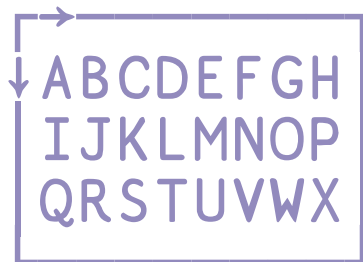
H



Data Transformation

Simple Indexing

- Choose Indexing
- Reach Indexing



$t \leftarrow 3 \quad 8 \rho \square A$

$t[(1 \quad 8)(2 \quad 7)]$

HO

$(1 \quad 8)(2 \quad 7) \text{ ? } t$

HO

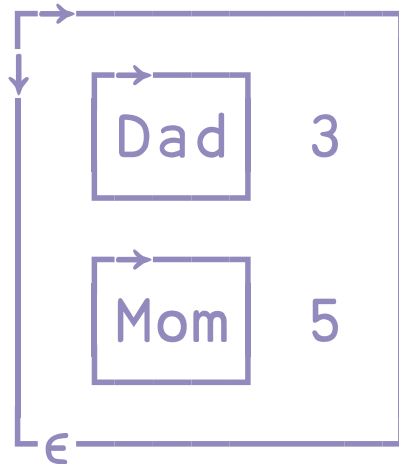


Data Transformation

Simple Indexing

Choose Indexing

● Reach Indexing



$s \leftarrow \text{'Dad' 'Mom' , } \bar{\tau} 3 \ 5$

$s[\epsilon(2 \ 1)3]$

m

$(2 \ 1)3 \triangleright s$

m

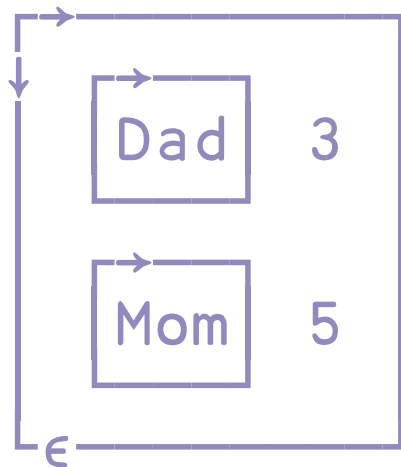


Data Transformation

Simple Indexing

Choose Indexing

Reach Indexing



```
s ← 'Dad' 'Mom', 3 5
```

$$s[(2\ 1)3][(1\ 1)2]$$

ma

$$((2\ 1)3)((1\ 1)2)?_S$$

ma



Data Transformation



Data Transformation

Select

$X \supseteq Y$



Indexing with Nested Vectors

3 Answers:

✧ $(\llbracket A \rrbracket) \llbracket \cdot \rrbracket \llbracket B \rrbracket$

✧ $\{B[\omega]\} \llbracket A \rrbracket$

✧ *Don't do that!*

Possibilities:

✧ $A \llbracket \cdot \rrbracket \llbracket \cdot \rrbracket \llbracket B \rrbracket$

✧ $\llbracket \cdot \rrbracket \circ B \llbracket \cdot \rrbracket \llbracket A \rrbracket$

✧ $A (\llbracket \cdot \rrbracket \rightarrow \llbracket \cdot \rrbracket) \llbracket B \rrbracket$

With Select $X \geq Y$:

✧ $A \geq \llbracket \cdot \rrbracket \llbracket B \rrbracket$

✧ $\geq \circ B \llbracket A \rrbracket$



Data Transformation

$X \supseteq Y$ Select

- Sort $\leftarrow (c \circ \Delta \square \vdash)$
- Sorts $\leftarrow \square \sim \circ c \circ \Delta \sim$ A "sort Y by X"
- Shuffle $\leftarrow (c \circ ? \sim \circ \neq \square \vdash)$
- Grade $\leftarrow ((c \text{ bounds } \circ \underline{1}) \square \text{ grades } \sim)$



Data Transformation

$X \triangleright Y$ Select

- Sort $\leftarrow (\uparrow \triangleright)$
- Sorts $\leftarrow \triangleright \circ \uparrow$ A "sort Y by X"
- Shuffle $\leftarrow (? \circ \neq \triangleright)$
- Grade $\leftarrow (\text{bounds} \circ \underline{\quad} \triangleright \text{grades})$



Data Transformation

$X \triangleright Y$ Select

- Sort $\leftarrow (\uparrow \triangleright \vdash)$
- Sorts $\leftarrow \triangleright \sim \circ \uparrow \sim$ a "sort Y by X"
- Shuffle $\leftarrow (? \sim \circ \neq \triangleright \vdash)$
- Grade $\leftarrow (\text{bounds} \circ \underline{\triangleright} \triangleright \text{grades} \sim)$



Data Transformation

$X \triangleright Y$ Select/Permute

- Sort $\leftarrow (\nabla \triangleright \vdash)$
- Sorts $\leftarrow \triangleright \sim \circ \nabla \sim$ a "sort Y by X"
- Shuffle $\leftarrow (? \sim \circ \neq \triangleright \vdash)$
- Grade $\leftarrow (\text{bounds} \circ \underline{\triangleright} \triangleright \text{grades} \sim)$

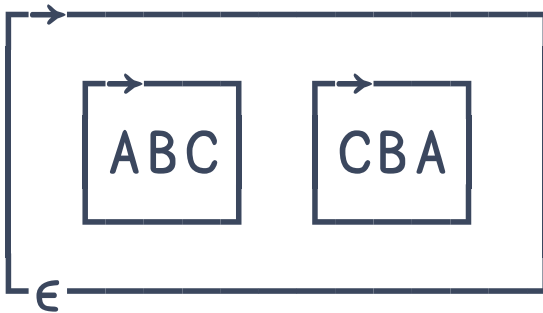


Indexing with Nested Vectors

I have

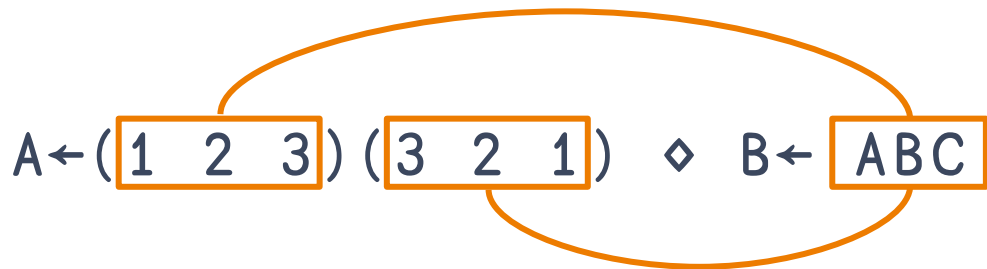
$A \leftarrow (1 \ 2 \ 3) (3 \ 2 \ 1) \diamond B \leftarrow 'ABC'$

I would like

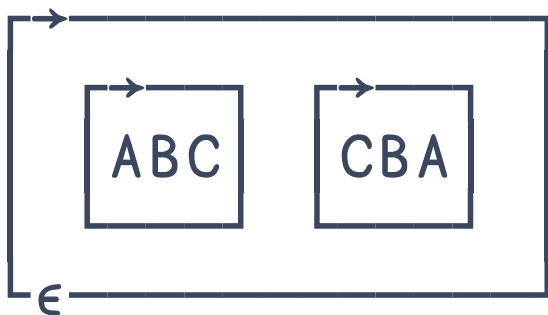


Indexing with Nested Vectors

I have

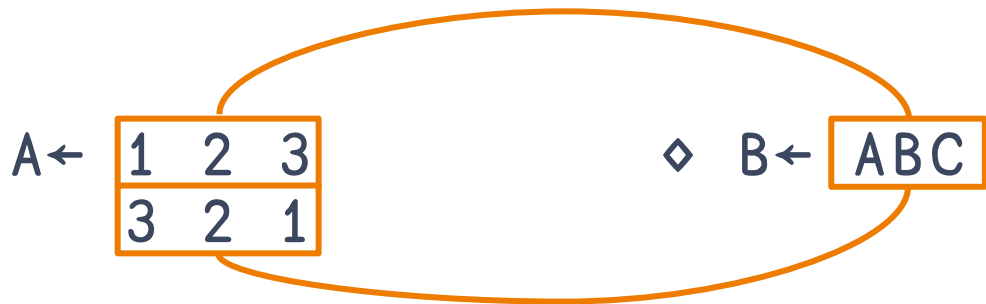


I would like

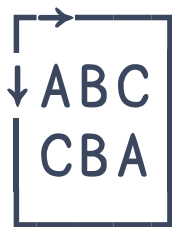


Indexing with Nested Vectors

I have

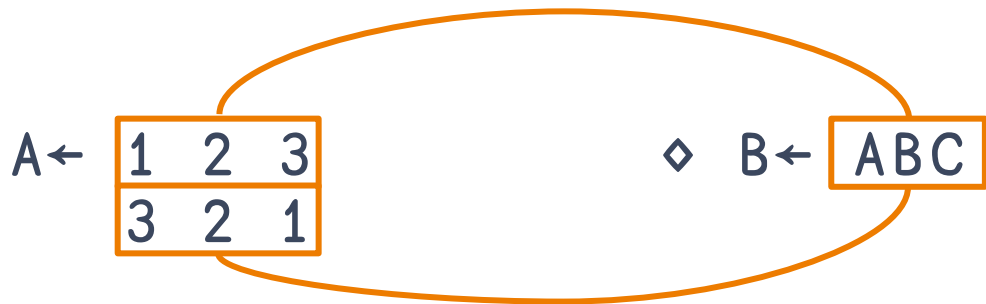


I would like



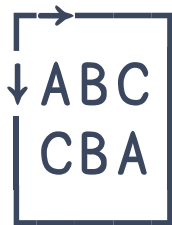
Indexing with Nested Vectors

I have



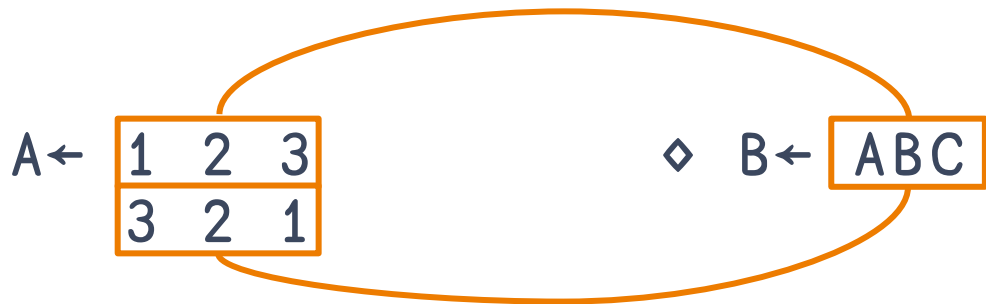
I would like

B[A]



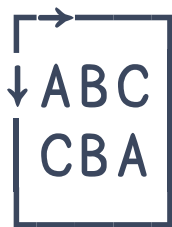
Indexing with Nested Vectors

I have



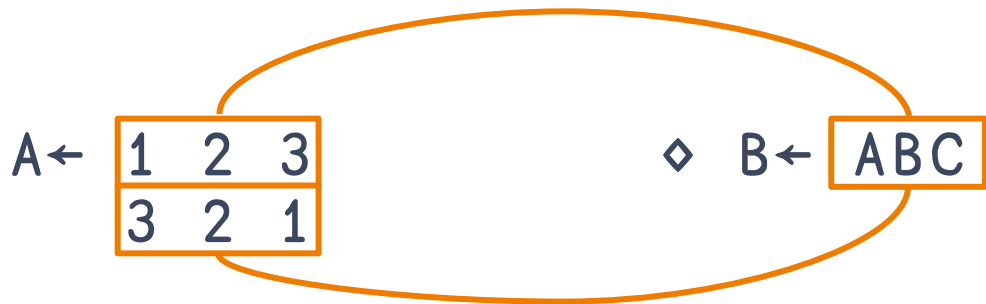
I would like

$$A \geq B$$



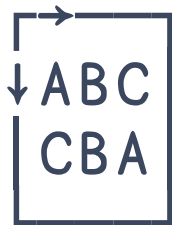
Indexing with Nested Vectors

I have



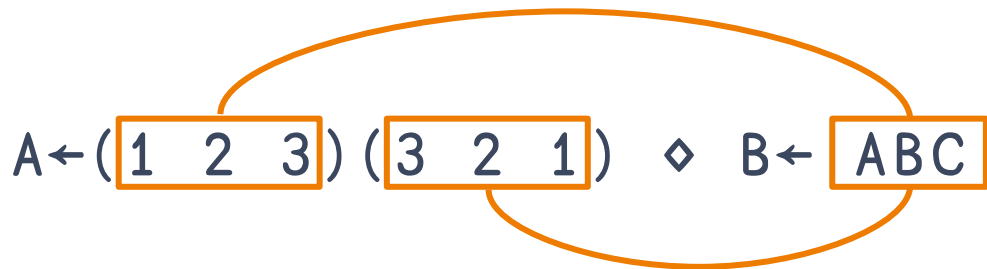
I would like

$$A(\geq 1)B$$



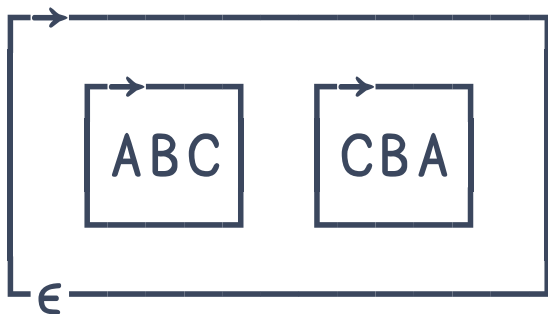
Function Application

I have



I would like

$A (\geq ? 1) B$



Function Application



Function Application

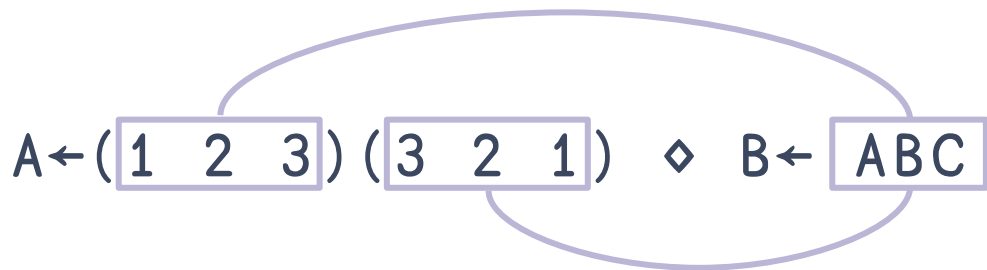
Depth

f ö k



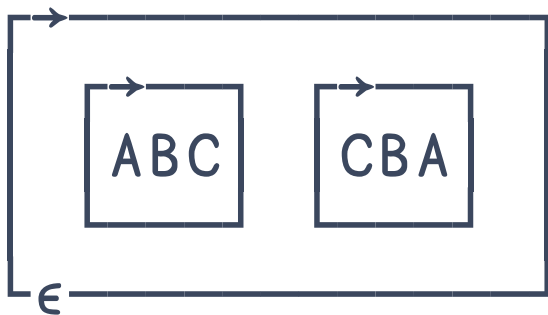
Function Application

I have



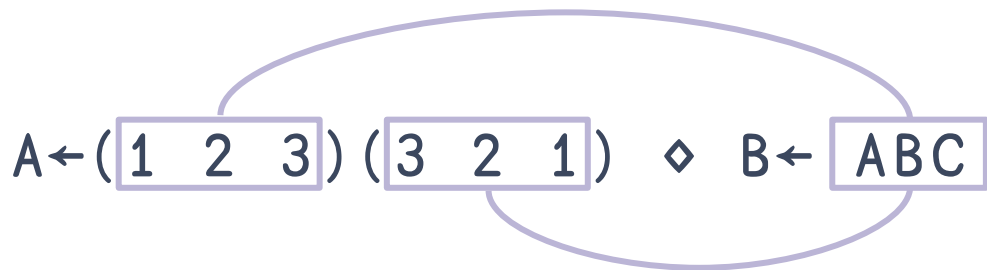
I would like

$A(\geq 1)B$

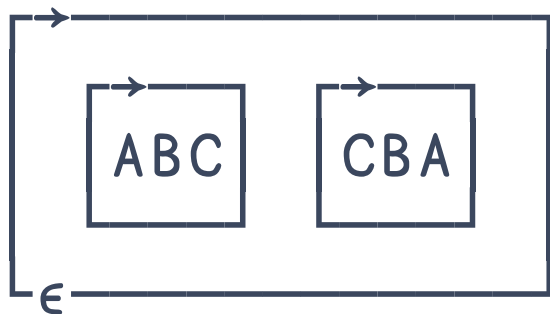


Function Application

I have



I would like



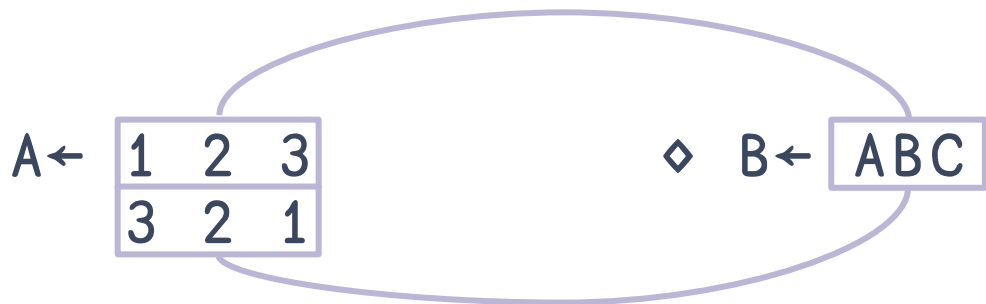
$A (\geq 1) B$

Watch this!

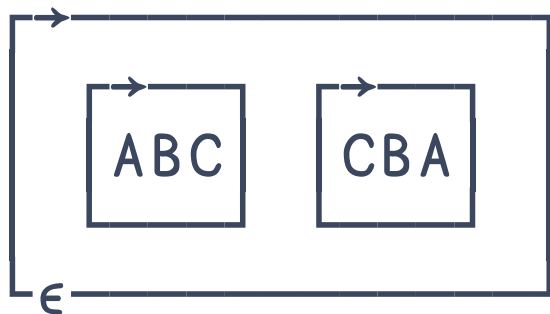


Function Application

I have



I would like



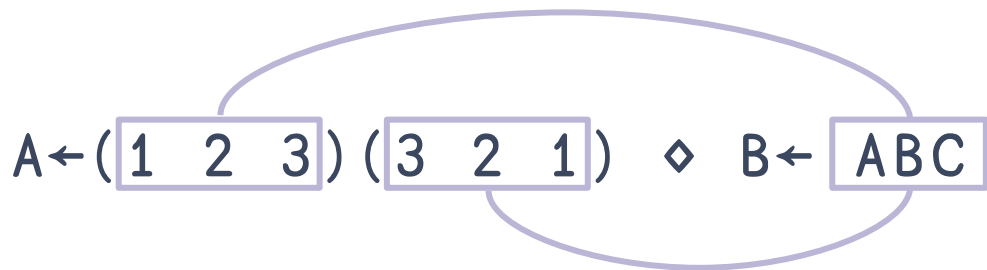
$A(\geq 1)B$

Watch this!

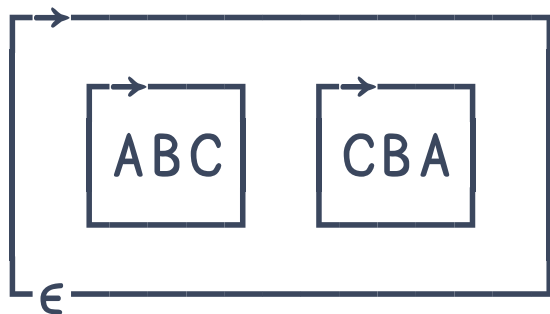


Function Application

I have



I would like



$A (\lambda x. 1) B$

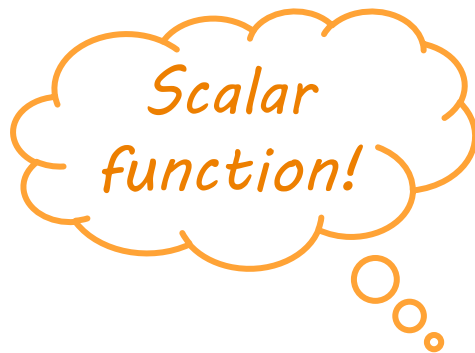
Watch this!



Function Application

Fp ← !
Fp 4 (5 6)

24	120 720
----	---------



Function Application

$Fd \leftarrow \{x / \iota \omega\}$

$Fd \ 4 \ (5 \ 6)$

DOMAIN ERROR

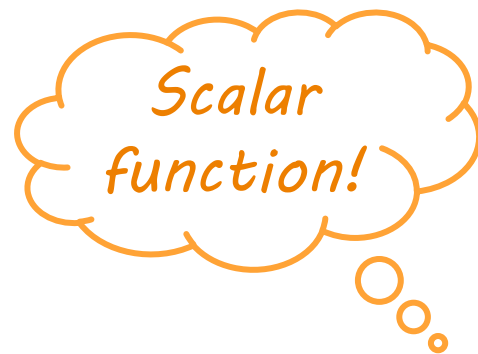
$Fd[0] \ Fd \leftarrow \{x / \iota \omega\}$

^

$Fs \leftarrow \{x / \iota \omega\} \ddot{o} 0$

$Fs \ 4 \ (5 \ 6)$

24	120 720
----	---------



Function Application

$Fd \leftarrow \{x / \iota \omega\}$

$Fd \ 4 \ (5 \ 6)$

DOMAIN ERROR

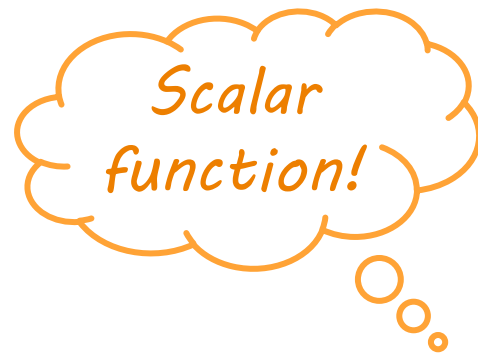
$Fd[0] \ Fd \leftarrow \{x / \iota \omega\}$

^

$Fs \leftarrow \{x / \iota \omega\} \ddot{o} 0$

$Fs \ 4 \ (5 \ 6)$

24	120 720
----	---------



Function Application

hi

$\square \leftarrow t1 \leftarrow 'hi'$



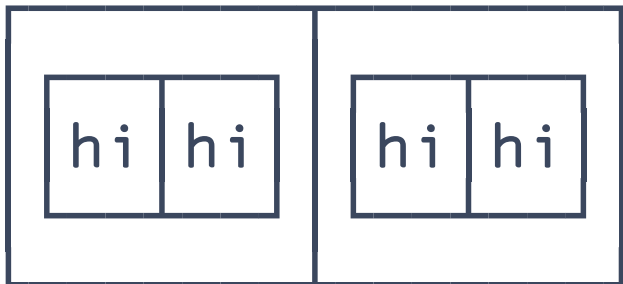
Function Application

$\square \leftarrow t2 \leftarrow 2\rho \subset t1 \leftarrow 'hi'$



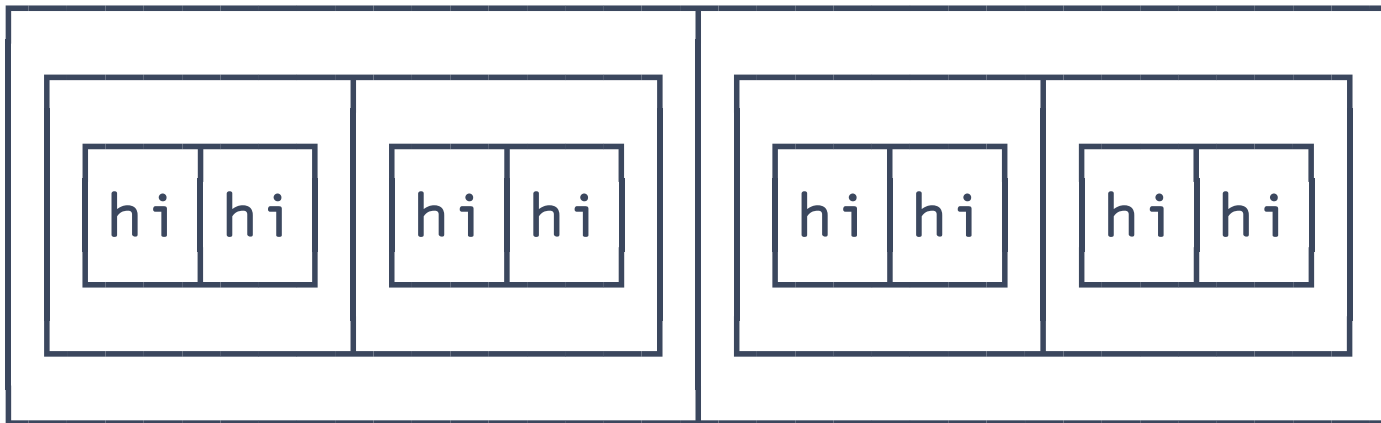
Function Application

$\square \leftarrow t3 \leftarrow 2\rho \subset t2 \leftarrow 2\rho \subset t1 \leftarrow 'hi'$



Function Application

$\square \leftarrow t4 \leftarrow 2\rho \subset t3 \leftarrow 2\rho \subset t2 \leftarrow 2\rho \subset t1 \leftarrow 'hi'$



Function Application

```
t4 ← 2 ρ ⊆ t3 ← 2 ρ ⊆ t2 ← 2 ρ ⊆ t1 ← 'hi '
```

```
' ^ . ' □ R ' \ u & ' ⊢ t1
```

Hi



Function Application

`t4 ← 2 ρ < t3 ← 2 ρ < t2 ← 2 ρ < t1 ← 'hi '`

`' ^ . ' □ R ' \ u & ' ⊢ t2`

Hi	Hi
----	----



Function Application

```
t4←2ρ<t3←2ρ<t2←2ρ<t1←'hi'
```

```
'^.'□R'\u&'⊢t3
```

DOMAIN ERROR: Invalid input source

```
'^.'□R'\u&'⊢t3
```

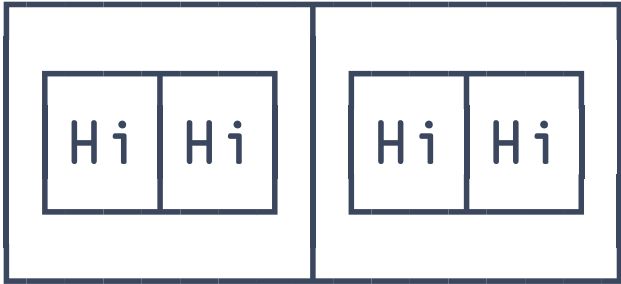
^



Function Application

$t_4 \leftarrow 2\rho \subset t_3 \leftarrow 2\rho \subset t_2 \leftarrow 2\rho \subset t_1 \leftarrow 'hi'$

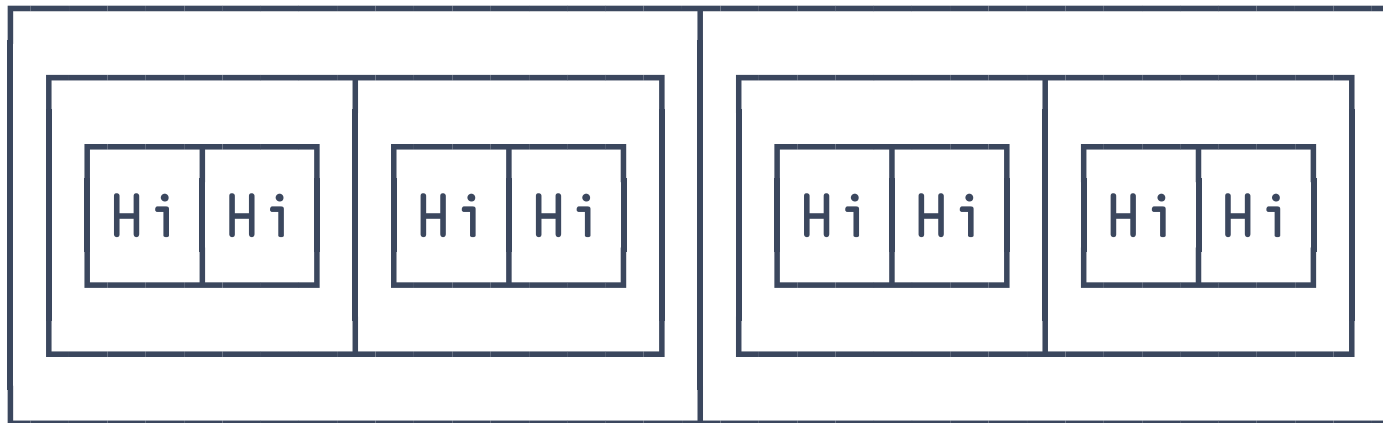
$'^{\wedge}.\square R'\backslash u\&'ö_2 \vdash t_3$



Function Application

$t_4 \leftarrow 2\rho \subset t_3 \leftarrow 2\rho \subset t_2 \leftarrow 2\rho \subset t_1 \leftarrow 'hi'$

$'^{\wedge}.\square R'\backslash u\&'ö_2 \vdash t_4$



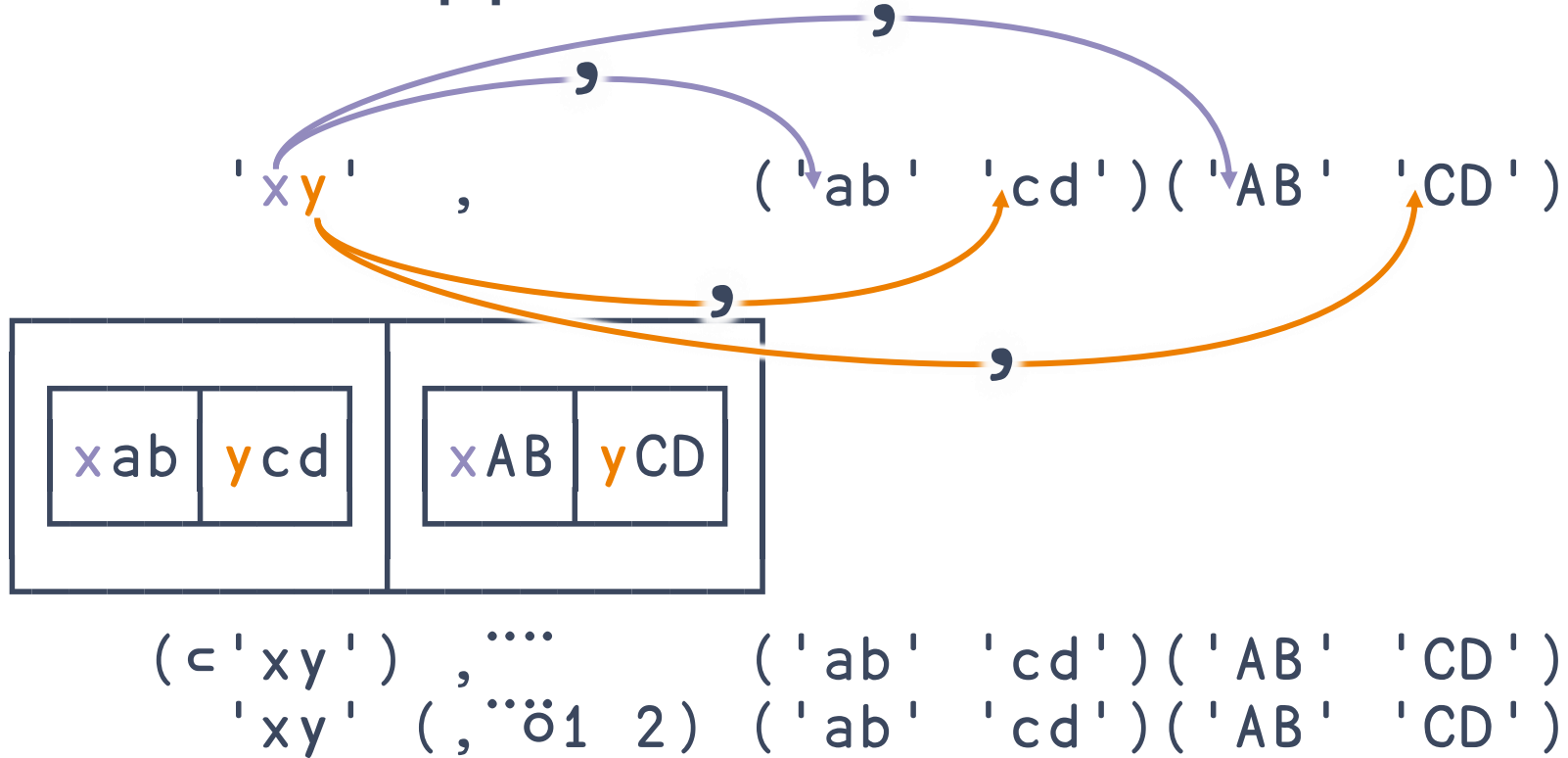
Function Application

$t_4 \leftarrow 2\rho \subset t_3 \leftarrow 2\rho \subset t_2 \leftarrow 2\rho \subset t_1 \leftarrow 'hi'$

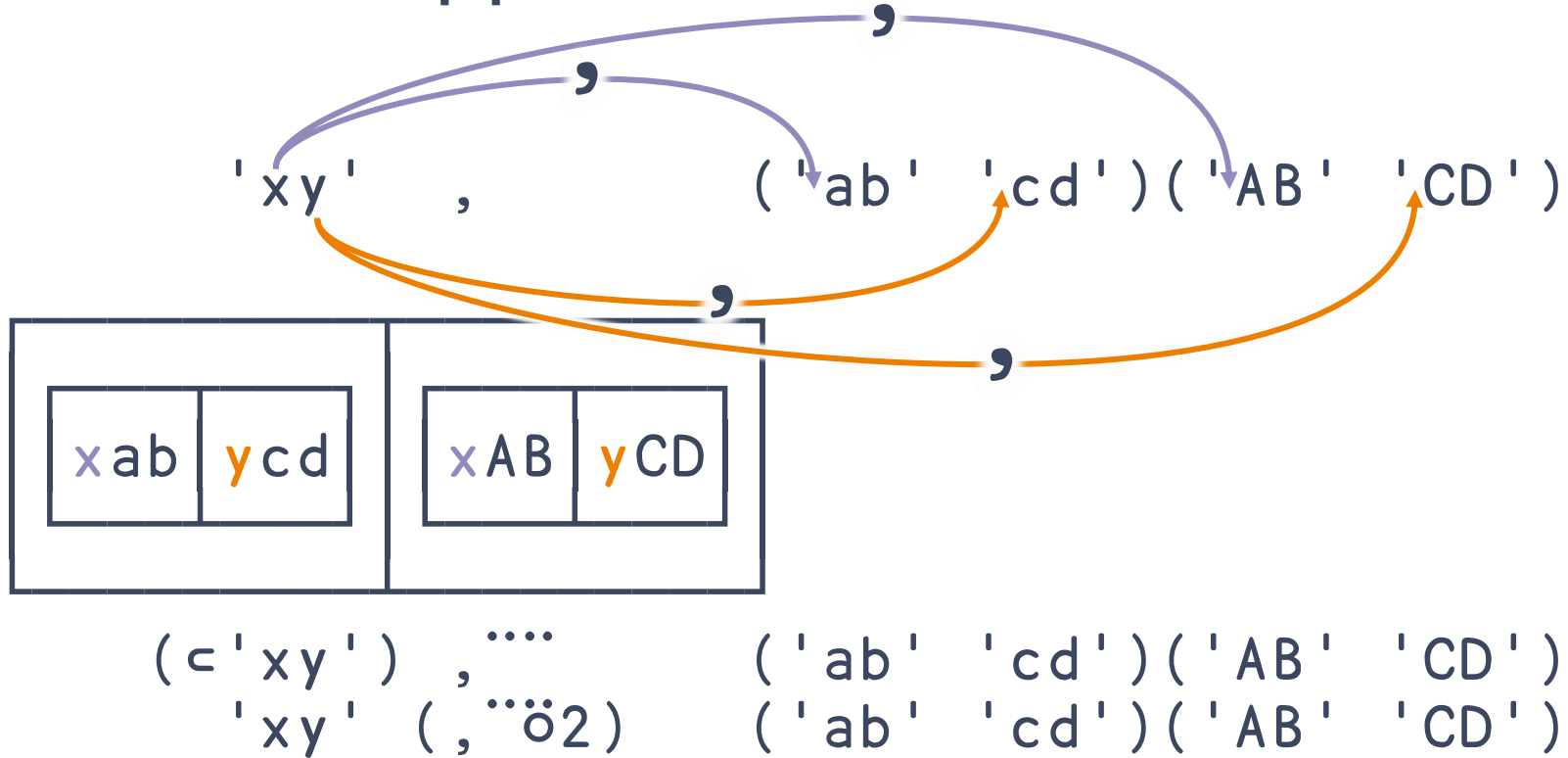
$'^{\wedge}.\ '\Box R'\ \backslash u\&' \ddot{o}2 \vdash t_4$
 $\{2 \leq | \equiv \omega : \nabla''\omega \ \diamond \ '^{\wedge}.\ '\Box R'\ \backslash u\&' \vdash \omega\}$



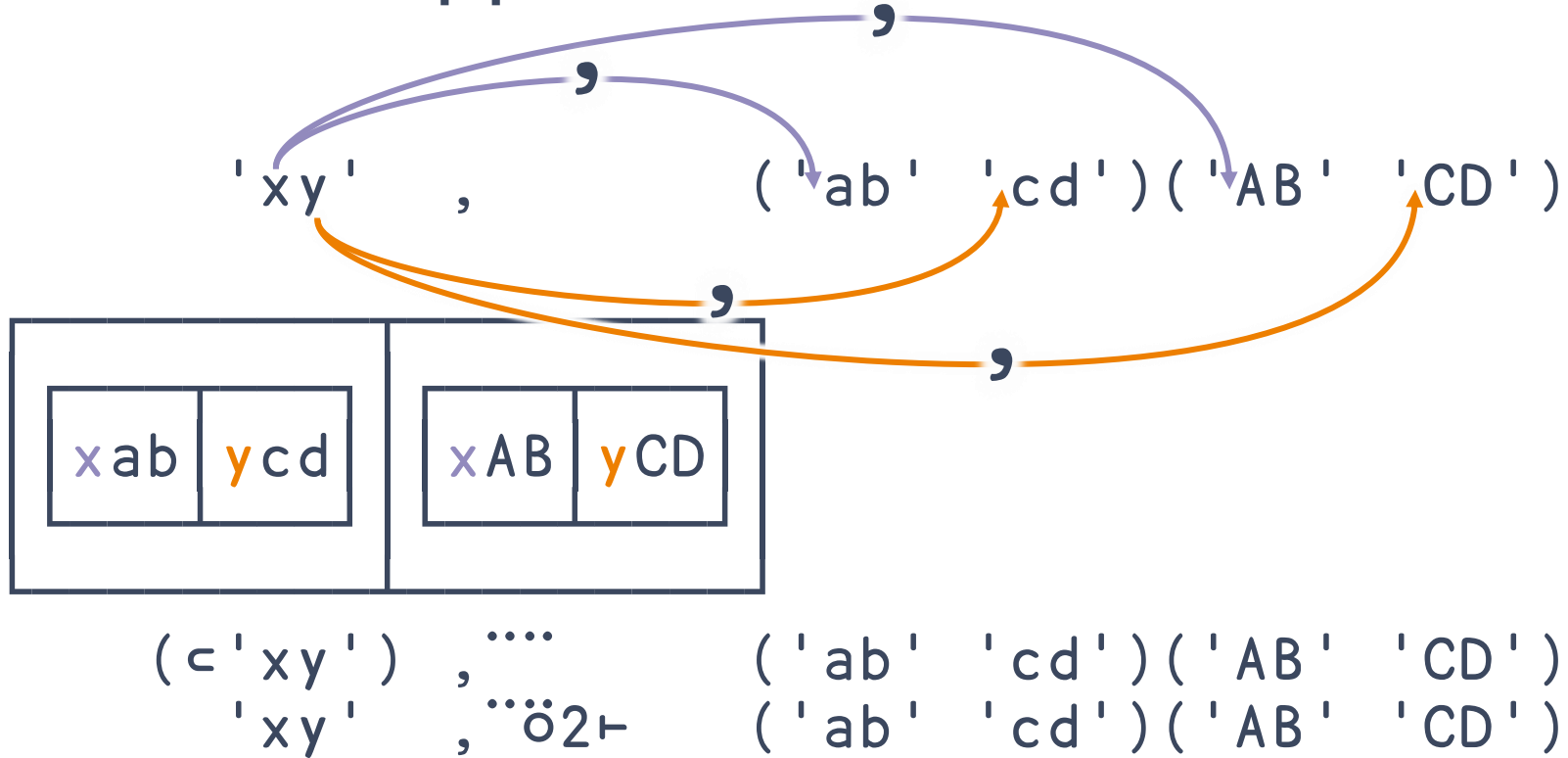
Function Application



Function Application



Function Application



Core Language

Data Transformation

Function Application

Function Composition

fök



Core Language

Data Transformation

$X \times Y$

ϕY

$X \sqcap Y$

$X \supseteq Y$

Function Application

$f \neq$

$f \ddot{*} g$

$f \ddot{o} k$

$f \ddot{o} k$

Function Composition

$f \ddot{o} g$

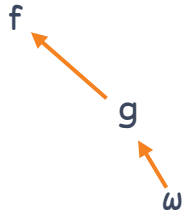
$f \ddot{o} g$

$f \circ g$

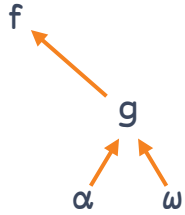


Function Composition

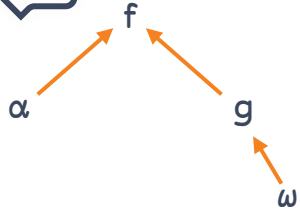
$f \circ g$ $f \circ g$ $f \circ g$



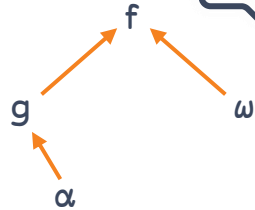
$f \circ g$



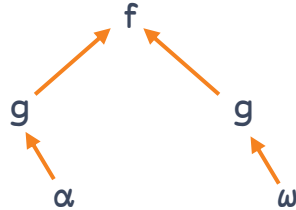
$f \circ g$



$f \circ g$

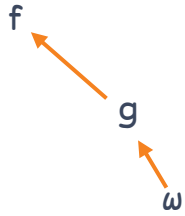


$f \circ g$

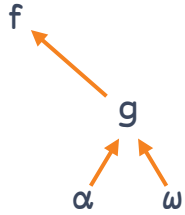


Function Composition

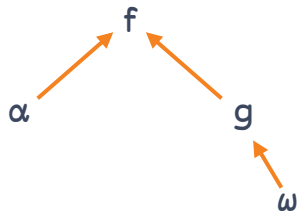
$f \circ g$ $f \circ g$ $f \circ g$



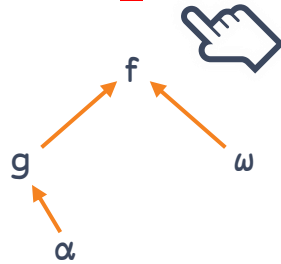
$f \circ g$



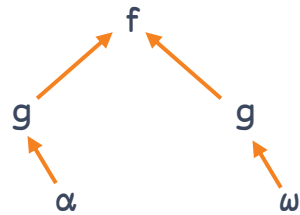
$f \circ g$



$f ? g$



$f \circ g$



Function Composition



Function Composition

Behind

$f \circ g$



Function Composition

Behind

$f \circ g$

$X (f \circ g) Y$



Function Composition

Behind

$f \circ g$

$X(f \quad)g \quad Y$



Function Composition

Behind

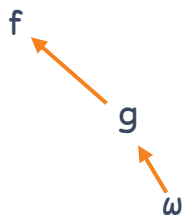
$f \circ g$

$(f \ X)g \ Y$

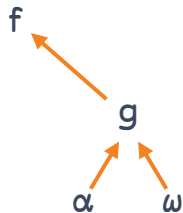


$f \circ g$ $f \circ g$ $f \circ g$

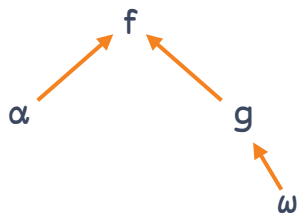
Function Composition



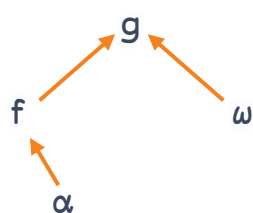
$f \circ g$



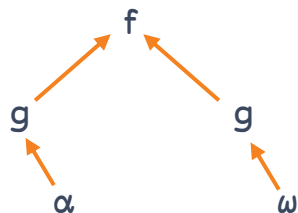
$f \circ g$



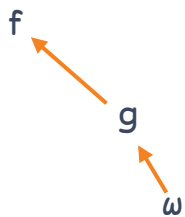
$f \circ g$



$f \circ g$

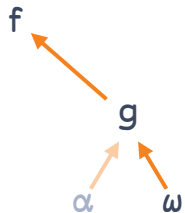


$f \circ g$ $f \circ g$ $f \circ g$

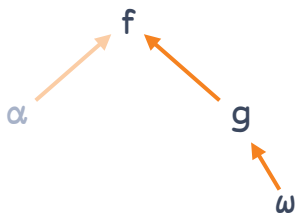


Function Composition

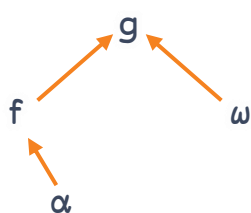
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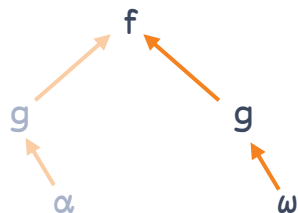
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$f \circ g$

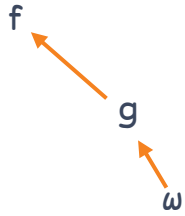


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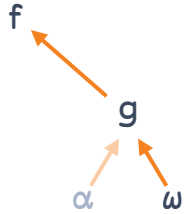


Function Composition

$f \circ g$ $f \circ g$ $f \circ g$

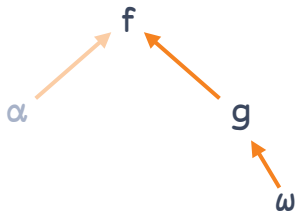


$f \circ g$

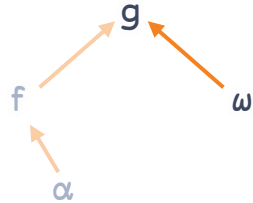


$$f \circ g \quad \omega \Leftrightarrow \quad g \quad \omega$$

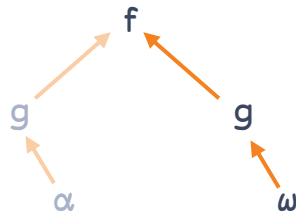
$f \circ g$



$f \circ g$

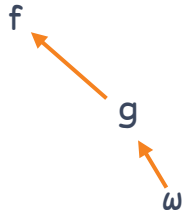


$f \circ g$

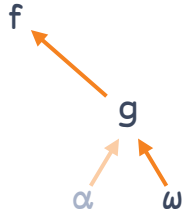


Function Composition

$f \circ g$ $f \circ g$ $f \circ g$

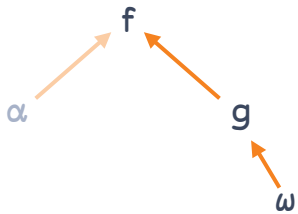


$f \circ g$

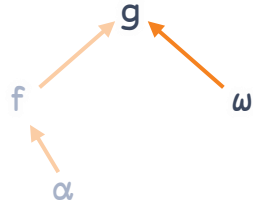


$$f \circ g \ \omega \Leftrightarrow g \ \omega$$

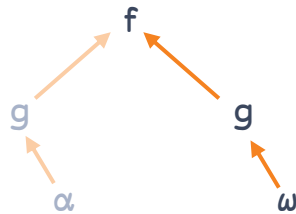
$f \circ g$



$f \circ g$

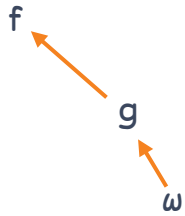


$f \circ g$

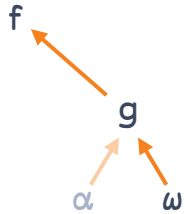


Function Composition

$f \circ \circ g$ $f \circ g$ $f \circ g$

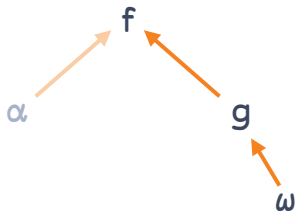


$f \circ \circ g$

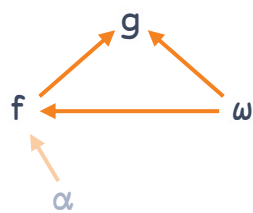


$$f \circ g \ \omega \Leftrightarrow f \circ g \circ \omega$$

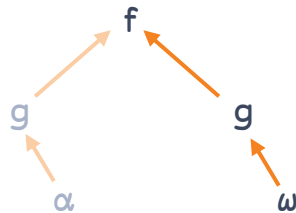
$f \circ g$



$f \circ g$



$f \circ g$





Essays/Hook Conjunction?

[< Essays](#)

Hook is a 2-train, an isolated sequence of two verbs, introduced in APL by K.E. Iverson and E.E. McDonnell, *Phrasal Forms*[↗](#), APL89, APL Quote-Quad, Volume 19, Number 4, 1989-08. It is defined as follows:

$$\begin{array}{l} (g\ h)\ y \quad \leftrightarrow \quad y\ g\ h\ y \\ x\ (g\ h)\ y \quad \leftrightarrow \quad x\ g\ h\ y \end{array}$$

For example, the monad `(=<.)` is a test for integers and `(+%)` computes a continued fraction -- `(+%)20$1` is an approximation of the golden ratio. Hook is based on the S combinator of [combinatory logic](#).

With over 17 years of hindsight, I believe it would have been better to use a conjunction (denoted by `h.`, say) to denote a hook rather than using a 2-train. Everything that can be done with the 2-train `(f g)` can be done with the conjunction `h.`, but `h.` does not require a special parsing rule.

The original motivation for assigning a meaning to a train of length 2 was so that a train of any length (greater than 1) would be interpreted: A train with odd length is a sequence of forks; a train with even length is either a hook (if of length 2) or a hook followed by a sequence of forks (if of length >2). Again with hindsight, the alternatives are:

0. Leave trains of even length uninterpreted -- just signal error.

1. Assign the "at" meaning to it:

$$\begin{array}{l} (g\ h)\ y \quad \leftrightarrow \quad g\ \ h\ y \\ x\ (g\ h)\ y \quad \leftrightarrow \quad g\ x\ h\ y \end{array}$$

That is, the [capped fork](#) meaning. You'd probably still have the capped fork. Compare:

```
[: f0 [: f1 f2 f3 f4
(f0 (f1 (f2 f3 f4)))
```

See also

- [Trains](#)

Contributed by [Roger Hui](#).

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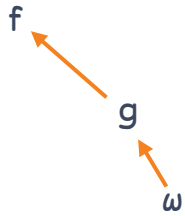
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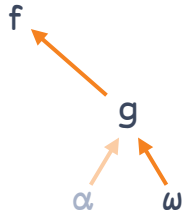
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Function Composition

$f \circ \circ g$ $f \circ g$ $f \circ g$

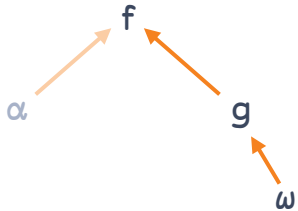


$f \circ \circ g$

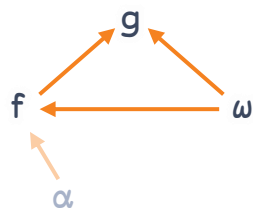


$$f \circ g \quad \omega \Leftrightarrow f \circ g \circ \omega$$

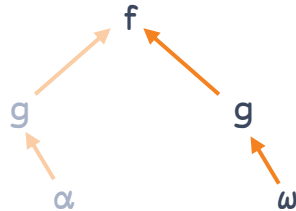
$f \circ g$



$f \circ g$

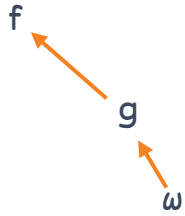


$f \circ \circ g$

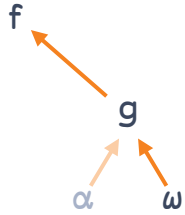


Function Composition

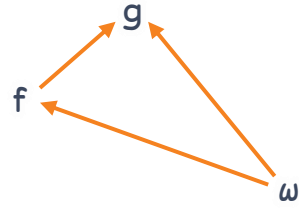
$f \circ g$ $f \circ g$ $f \circ g$



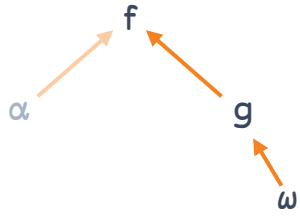
$f \circ g$



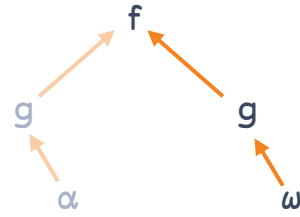
$f \circ g$



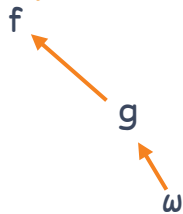
$f \circ g$



$f \circ g$

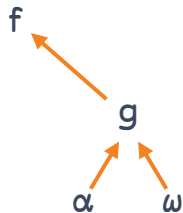


$f \ddot{o} g$ $f \circ g$ $f \ddot{o} g$
pre/postprocess

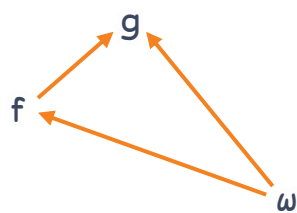


Function Composition

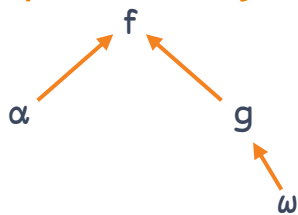
$f \ddot{o} g$
postprocess



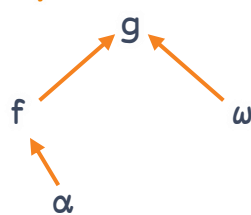
$f \circ g$
preprocess left



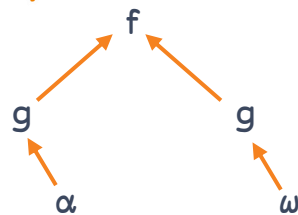
$f \circ g$
preprocess right



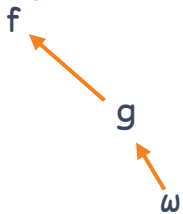
$f \circ g$
preprocess left



$f \ddot{o} g$
preprocess both

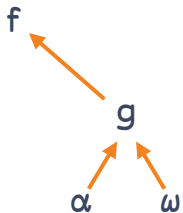


$f \ddot{o} g$ $f \circ g$ $f \ddot{o} g$
pre/postprocess

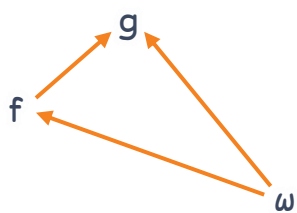


Function Composition

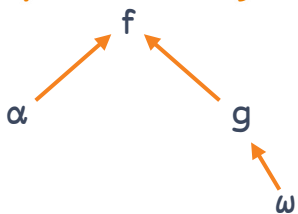
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postprocess



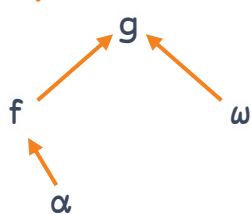
$f \circ g$
preprocess left



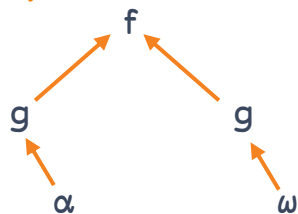
$f \circ g$
preprocess right



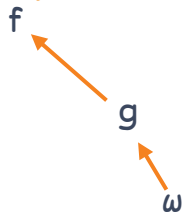
$f \circ g$
preprocess left



$f \ddot{o} g$
preprocess both

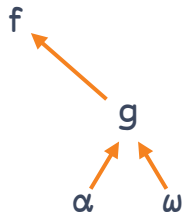


$f \ddot{o} g$ $f \circ g$ $f \ddot{o} g$
pre/postprocess

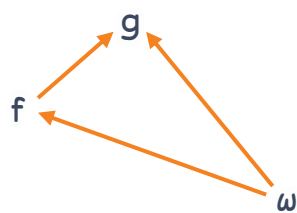


Function Composition

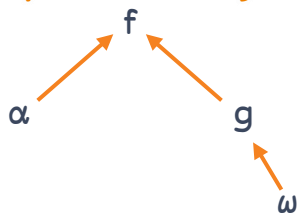
$f \ddot{o} g$
postprocess



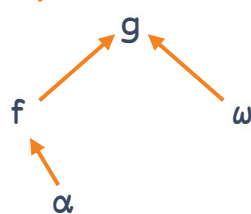
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preprocess left



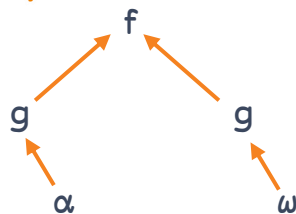
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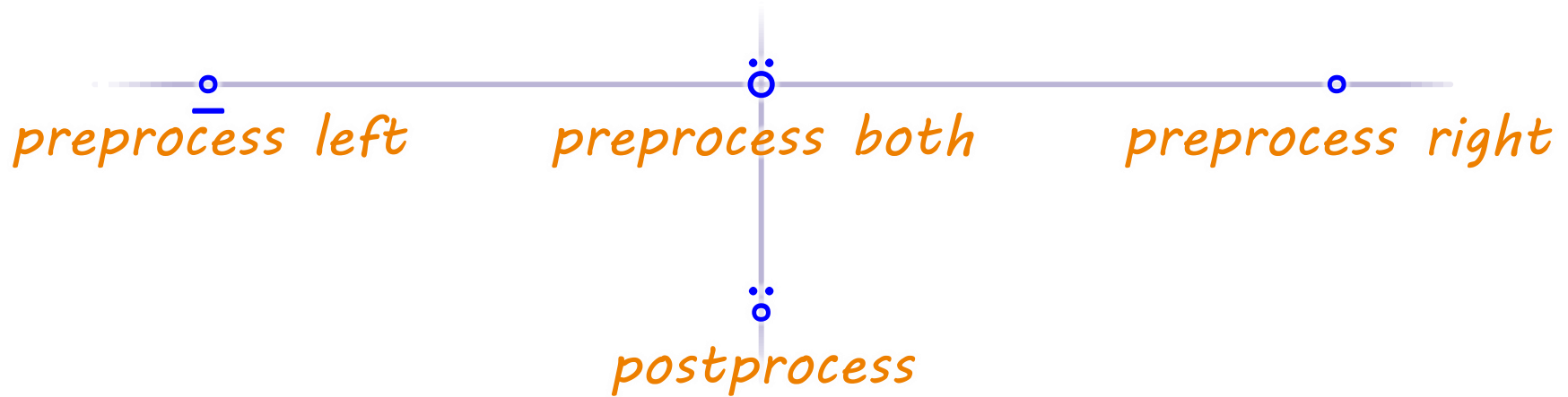
$f \circ g$
preprocess left



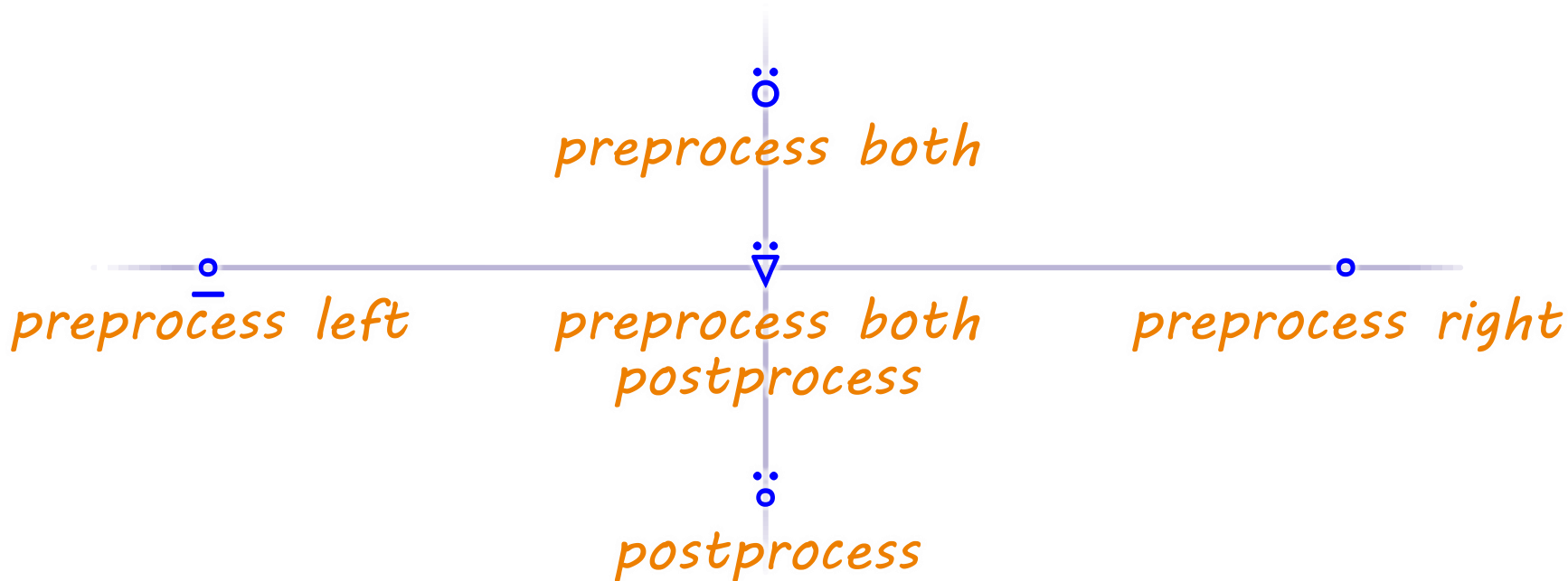
$f \ddot{o} g$
preprocess both



Function Composition



Function Composition



Function Composition

$f \circ g$ Behind with $X \supseteq Y$ Select/Permute

Sort $\leftarrow (\lambda \supseteq \vdash)$



Function Composition

$f \circ g$ Behind with $X \supseteq Y$ Select/Permute

Sort $\leftarrow \downarrow \circ \supseteq$



Function Composition

$f \circ g$ Behind with $X \supseteq Y$ Select/Permute

Sort $\leftarrow \downarrow \circ \supseteq$

Sorts $\leftarrow \supseteq \circ \downarrow$ A "sort Y by X"



Function Composition

$f \circ g$ Behind with $X \supseteq Y$ Select/Permute

Sort $\leftarrow \lambda \circ \supseteq$

Sorts $\leftarrow \lambda \circ \supseteq$ A "sort Y by X"



Function Composition

`f_g` Behind with `X ≥ Y` Select/Permute

`Sort` ← `⋄_o_≥`

`Sorts` ← `⋄_o_≥` a "sort Y by X"

✧ `Shuffle` ← `(?~o≠_≥_+)`



Function Composition

$f \circ g$ Behind with $X \supseteq Y$ Select/Permute

Sort $\leftarrow \lambda \circ \supseteq$

Sorts $\leftarrow \lambda \circ \supseteq$ A "sort Y by X"

Shuffle $\leftarrow ? \circ \neq \circ \supseteq$



Function Composition

$f \circ g$ Behind

- SameAsFirst $\leftarrow \supset \circ =$
- HasDuplicates $\leftarrow \cup \circ \equiv$
- Palindrome $\leftarrow \phi \circ \equiv$
- IsPermutation $\leftarrow \Delta \circ \Delta \circ \equiv$



Function Composition

$f_{\circ g}$ Behind

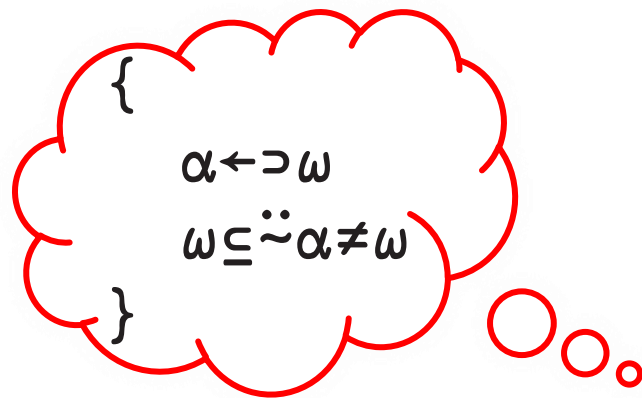
Integer $\leftarrow \lfloor _ \rceil =$

Split $\leftarrow (\supset _ \neq \subseteq \vdash)$

Scale $\leftarrow \lceil / _ (\div \sim) \rceil$

Deviation $\leftarrow (+ \neq \div \neq) _ (- \sim)$

'/' ($\neq \subseteq \vdash$) 'hi/how/goes'
($\supset _ \neq \subseteq \vdash$) '/hi/how/goes'
'/' ($\supset _ \neq \subseteq \vdash$) '/hi/how/goes'



Function Composition

`f_g` Behind

```
Filters ← of
>◦5 Filters 2 7 1 8 2 8
7 8 8
φ◦≡ ``Filters 'racecar' 'racer' 'toot'
racecar toot
```



Function Composition

$f \circ g$ Behind

```

      >◦5  ◦/ 2 7 1 8 2 8
7 8 8
      ϕ◦≡◦/ 'racecar' 'racer' 'toot'
racecar  toot
```



Function Composition

$f \circ g$ Behind

- $\text{Whence} \leftarrow \iota \circ \epsilon \quad \mathcal{A} \quad \{(\iota \alpha) \in \omega\}$
- $\text{InPoly} \leftarrow \bar{\gamma} \circ \perp \quad \mathcal{A} \quad \{(\bar{\gamma} \alpha) \perp \omega\}$
- $\text{Shapes} \leftarrow \rho \circ \rho \quad \mathcal{A} \quad \{(\rho \alpha) \rho \omega\}$
- $\text{ToFile} \leftarrow \mathsf{c} \circ \square \text{NPUT} \quad \mathcal{A} \quad \{(\mathsf{c} \alpha) \square \text{NPUT} \ \omega\}$



Function Composition

f_og Behind

FCat $\leftarrow \overline{\phi}_o$,

RIndex $\leftarrow \Theta_o \wr$

RDrop $\leftarrow -_o \downarrow$

RndSfx $\leftarrow -\ddot{o} ?_o \uparrow$

APL\360: 'abc';42;'def'

Dyalog: 'abc',($\overline{\phi}$ 42),'def'

20.0: 'abc',42 $\overline{\phi}_o$, 'def'

$\mathcal{A} \{(\Theta\alpha)\wr\omega\}$

$\mathcal{A} \{(-\alpha)\downarrow\omega\}$

$\mathcal{A} \{(-?\alpha)\uparrow\omega\}$



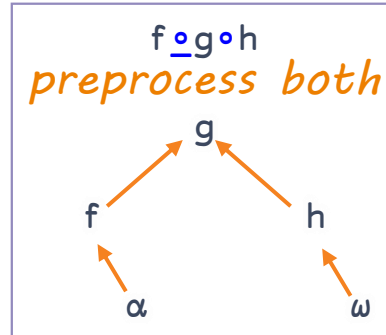
Function Composition

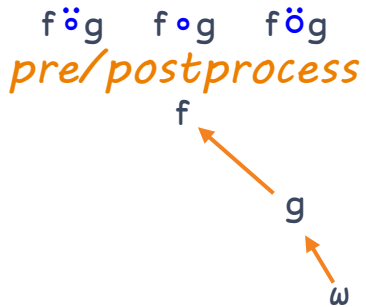
$f \circ g$ Behind

Split-compose $X \quad f \circ g \circ h \quad Y$

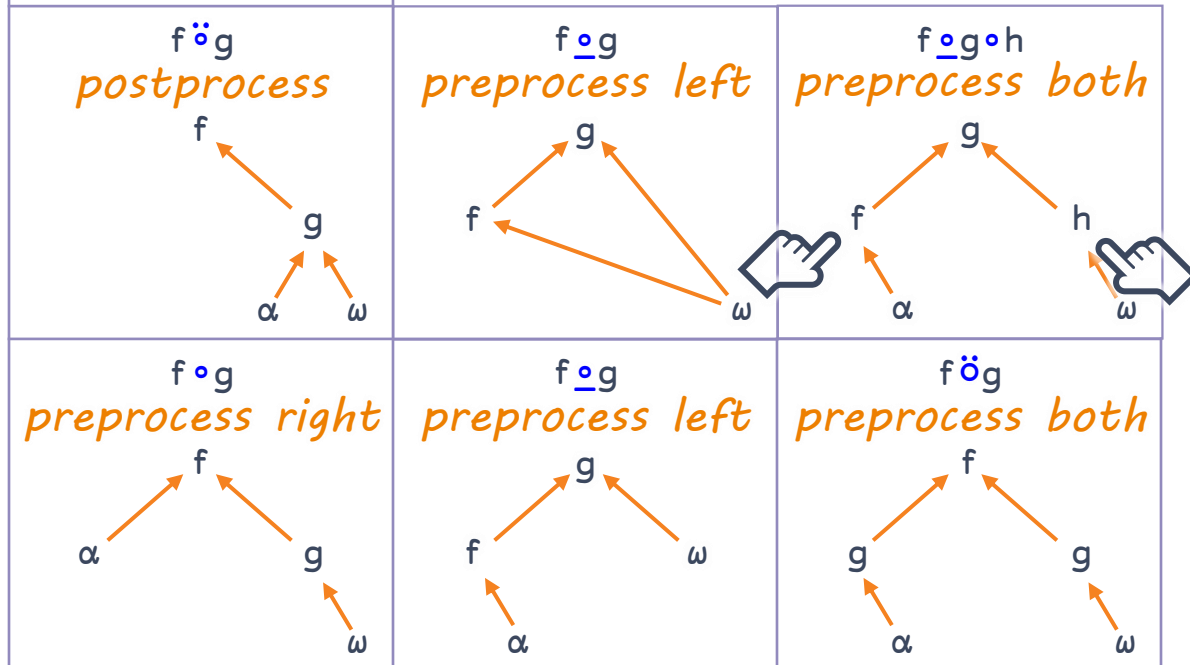


Function Composition

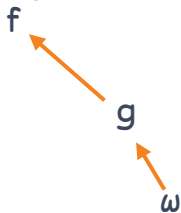




Function Composition

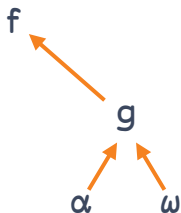


$f \ddot{o} g$ $f \circ g$ $f \ddot{o} g$
pre/postprocess

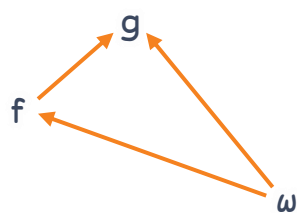


Function Composition

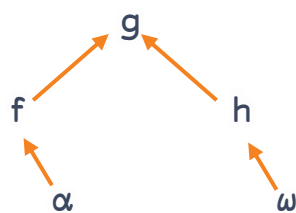
$f \ddot{o} g$
postprocess



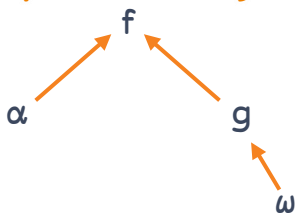
$f \circ g$
preprocess left



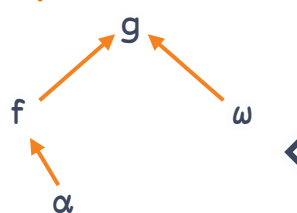
$f \circ g \circ h$
preprocess both



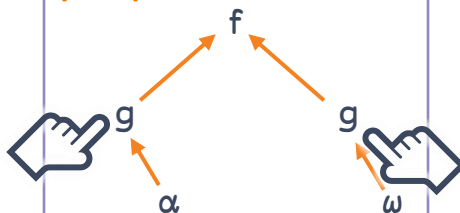
$f \circ g$
preprocess right



$f \circ g$
preprocess left



$f \ddot{o} g$
preprocess both



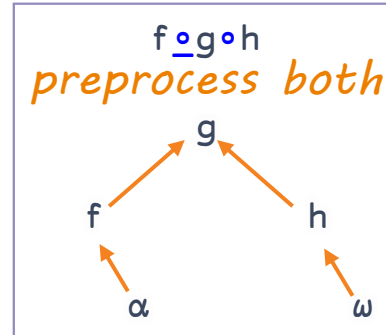
$$\alpha \quad g \circ f \circ g \quad \omega$$

$$\Leftrightarrow$$

$$\alpha \quad f \ddot{o} g \quad \omega$$



Function Composition



Function Composition

$f \circ g$ Behind

Split-compose

$X \quad f \circ g \circ h \quad Y$

Pre-18.0: $g \circ f \circ h$
18.0: $(f \circ g \circ h)$
20.0: $f \circ g \circ h$



Function Composition

$f \circ g$ Behind

Split-compose

$X \quad f \circ g \circ h \quad Y$

Pre-18.0: $((f \dashv)g(h \vdash))$

18.0: $(f \ddot{\circ} \dashv \quad g \quad h \ddot{\circ} \vdash)$

20.0: $f \circ g \circ h$



Function Composition

$f \circ g$ Behind

Split-compose

$X \quad f \circ g \circ h \quad Y$

Hybrid mitigation

$2 \circ \mid \quad \circ /$

Pre-18.0: $(2 \circ \mid \{ \alpha / \omega \} \vdash)$

18.0: $(2 \circ \mid \vdash \ddot{\circ} / \vdash)$

20.0: $2 \circ \mid \quad \circ /$



Language Enhancements

Data Transformation

Function Application

Function Composition

f_og



Language Enhancements

Data Transformation

$X \times Y$

ϕY

$X \sqcap Y$

$X \supseteq Y$

Function Application

$f \neq$

$f \ddot{*} g$

$f \ddot{o} k$

$f \ddot{o} k$

Function Composition

$f \ddot{o} g$

$f \ddot{o} g$

$f \circ g$

$f \underline{o} g$



Language Enhancements

Data Transformation

Select

$Y[X; ;]$

$X \supseteq Y$

Function Application

Depth

20.0

$X \text{ f } \ddot{o} k \ Y$

Function Composition

Behind

$(f \ X) g \ Y$

$X \text{ f } \underline{o} g \ Y$



Language Enhancements

Data Transformation

Select

$Y[X; ;]$

$X \supseteq Y$

Function Application

Depth

20.0

$X \text{ f } \ddot{o} k \ Y$

Function Composition

Behind

$(f \ X) g \ Y$

$X \text{ f } \underline{o} g \ Y$

Axis Manipulation



Axis Manipulation

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$A \times \subset B$

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32

$\bar{;} \neq A \times \subset B$

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32

$, / \bar{;} \neq A \times \subset B$

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32

$\supset , / \bar{;} \neq A \times \subset B$

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32



Axis Manipulation

$$\begin{bmatrix} 5 & 6 & 10 & 12 \\ 7 & 8 & 14 & 16 \\ 15 & 18 & 20 & 24 \\ 21 & 24 & 28 & 32 \end{bmatrix}$$

$A \times c B$

5 6	10 12
7 8	14 16
15 18	20 24
21 24	28 32



Axis Manipulation

A \circ . \times B

5	6
7	8

10	12
14	16

5	6
7	8

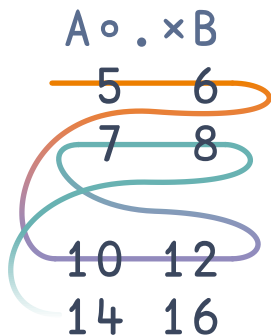
10	12
14	16

15	18
21	24

20	24
28	32



Axis Manipulation



5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32

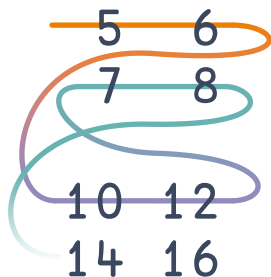
15 18
21 24

20 24
28 32



Axis Manipulation

1 2 3 4 ϕ A \circ . \times B



15 18
21 24

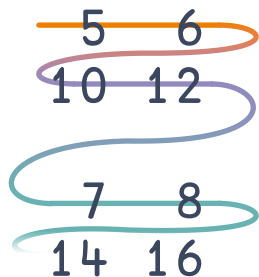
20 24
28 32

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32



Axis Manipulation

1 3 2 4 ϕ A \circ . \times B



15 18
20 24

21 24
28 32

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32



Axis Manipulation

, [1 2] 1 3 2 4 A . × B

5 6
10 12

7 8
14 16

15 18
20 24

21 24
28 32

$$\begin{bmatrix} 5 & 6 & 10 & 12 \\ 7 & 8 & 14 & 16 \\ 15 & 18 & 20 & 24 \\ 21 & 24 & 28 & 32 \end{bmatrix}$$


Axis Manipulation

,[2 3] ,[1 2] 1 3 2 4 A . × B

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32



Axis Manipulation

, [1 2] * -1 , [1 2] 1 3 2 4 * A * . * B

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32



Axis Manipulation

,[1 2]÷2 ,[1 2] 1 3 2 4 A . × B

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32

5	6	10	12
7	8	14	16
15	18	20	24
21	24	28	32



Axis Manipulation

$\begin{bmatrix} ? & 2 & ? & 1 & 3 & 2 & 4 \end{bmatrix} \circ A \cdot B$
 $\begin{bmatrix} 5 & 6 & 10 & 12 \\ 7 & 8 & 14 & 16 \\ 15 & 18 & 20 & 24 \\ 21 & 24 & 28 & 32 \end{bmatrix}$

$$\begin{bmatrix} 5 & 6 & 10 & 12 \\ 7 & 8 & 14 & 16 \\ 15 & 18 & 20 & 24 \\ 21 & 24 & 28 & 32 \end{bmatrix}$$


Axis Manipulation



Axis Manipulation

Demote

v Y



Axis Manipulation

Promote

$\wedge Y$

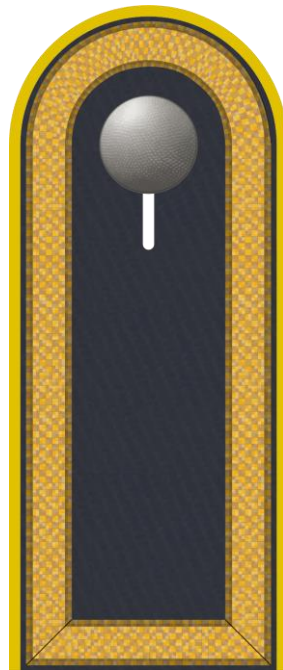


Axis Manipulation

'H a l l o '

Promote

^ Y



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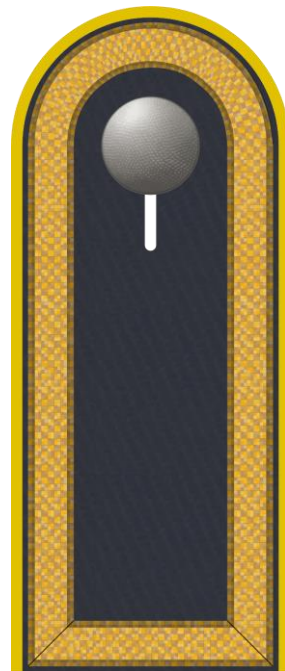


Axis Manipulation

5 ρ 'Hallo'

Promote

^ Y



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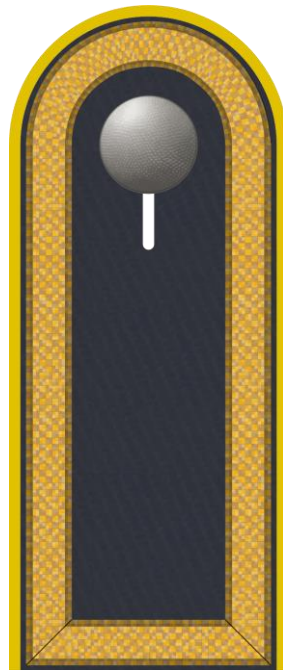


Axis Manipulation

1 ≠p 'Hal lo'

Promote

^ Y



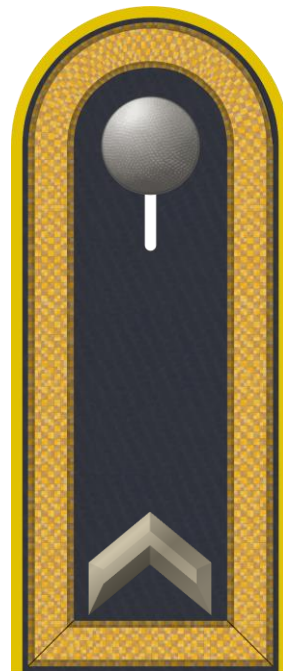
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Axis Manipulation

2 $\neq p \wedge \text{'Hallo'}$

Promote



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Axis Manipulation

3 $\neq p \wedge \wedge 'H a l l o '$

Promote



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Axis Manipulation

^ 'Hallo'

□ML>1: ➤

⊘; 'Hallo'

↑, < 'Hallo'

1 5ρ 'Hallo'

, [÷2] 'Hallo'

, [□IO-÷2] 'Hallo'

'Hallo';ö^ 'Welt!'

Hallo
Welt!

↑ 'Hallo' 'Welt!'

□ML>1: ➤



Axis Manipulation

^ 'Hallo'

✓ , [i2]

, [f2]

'Hallo'; ö ^ 'Welt!'

Hallo

Welt!

⊘; 'Hallo'

↑, < 'Hallo'

1 5ρ 'Hallo'

^ , [÷2] 'Hallo'

, [⊠IO-÷2] 'Hallo'

↑ 'Hallo' 'Welt!'



Axis Manipulation

^ 'Hallo'

✓ , [ι2]

, [ι̇2]

'Hallo' ; ö ^ 'Welt!'

Hallo

Welt!

⊘ , 'Hallo'

↑ , < 'Hallo'

1 5ρ 'Hallo'

^ , [÷2] 'Hallo'

, [⊠IO-÷2] 'Hallo'

↑ 'Hallo' 'Welt!'



Axis Manipulation

^ 'Hallo'

v , [ι2]

, [ι̇2]

'Hallo' ; ö ^ 'Welt!'

Hallo

Welt!

⊘ ; 'Hallo'

↑ , < 'Hallo'

1 5ρ 'Hallo'

^ , [÷2] 'Hallo'

, [⊠IO-÷2] 'Hallo'

↑ 'Hallo' 'Welt!'



Axis Manipulation

```
      (v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      |
```



Axis Manipulation

```
      (v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      |(v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0
```



Axis Manipulation

```
      (v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      b |(v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0
```



Axis Manipulation

```
      (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ; ö ^ |(v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
```



Axis Manipulation

```

      (v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      b ; ö ^ (v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0
0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      |(v\b) ^ φ v \ φ b ← 0  1  0  0  1  1  0  0

```



Axis Manipulation

```

      (v\b) ^ phi v \ phi b <- 0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      b ; ö ^ (v\b) ^ phi v \ phi b <- 0  1  0  0  1  1  0  0
0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      b | (v\b) ^ phi v \ phi b <- 0  1  0  0  1  1  0  0

```



Axis Manipulation

```

      (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ; ö ^ (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ( |(v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0

```



Axis Manipulation

```

      (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ; ö ^ (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ( (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0 |

```



Axis Manipulation

```

      (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ; ö ^ (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ( (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0 ) |
  
```



Axis Manipulation

```

      (v\b) ^ phi v \ phi b <- 0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ; ö ^ (v\b) ^ phi v \ phi b <- 0 1 0 0 1 1 0 0
0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      | b ( (v\b) ^ phi v \ phi b <- 0 1 0 0 1 1 0 0 )

```



Axis Manipulation

```

      (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      b ; ö ^ (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0
0 1 0 0 1 1 0 0
0 1 1 1 1 1 0 0
      ↑ | b ( (v\b) ^ φ v \ φ b ← 0 1 0 0 1 1 0 0 )

```



Axis Manipulation

```

      (v\b) ^ phi v \ phi b <- 0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      b ; ö ^ (v\b) ^ phi v \ phi b <- 0  1  0  0  1  1  0  0
0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      ↑ b ( (v\b) ^ phi v \ phi b <- 0  1  0  0  1  1  0  0 )
0  1  0  0  1  1  0  0
0  1  1  1  1  1  0  0
      |

```



Language Enhancements

Data Transformation

Function Application

Function Composition

Axis Manipulation

$\vee Y$ $\wedge Y$



Language Enhancements

Data Transformation	Select	$Y[X; ;]$	$X \supseteq Y$
Function Application	Depth	$X \text{ f } \cdots \subset \subset Y$	$X \text{ f } \ddot{o} k Y$
Function Composition	Behind	$(\text{f } X) g Y$	$X \text{ f } \underline{o} g Y$
Axis Manipulation	De/Promote	$, [\overset{!}{\div} 2] Y$	$\vee Y \quad \wedge Y$

Questions?

