APL on GPUs – A Progress Report with a Touch of Machine Learning

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@ Dyalog'17, Elsinore

Motivation

Goal:
High-performance at the fingertips of domain experts.

Why APL:
APL provides a ***powerful and concise*** notation for array operations.

APL programs are inherently parallel - not just parallel, but ***data-parallel***.

There is lots of APL code around - some of which is looking to run faster!

Challenge:
APL is dynamically typed. To generate efficient code, we need ***type inference***:

- Functions are *rank-polymorphic*.
- Built-in operations are overloaded.
- Some *subtyping* is required (e.g., any integer 0,1 is considered boolean).

Type inference algorithm compiles APL into a *typed array intermediate language* called **TAIL** (ARRAY'14).

APL Supported Features

Dfns-syntax for functions and operators (incl. trains).

```
else ← {(αα⍣α)(ωω⍣(~α))ω}

mean ← +/÷≢
```

Dyalog APL compatible built-in operators and functions (limitations apply).

Scalar extensions, identity item resolution, overloading resolution.

Limitations:

- Static scoping and static rank inference
- Limited support for nested arrays
- Whole-program compilation
- No execute!

TAIL - as an IL

APL → **TAIL** → Futhark

- Type system ***expressive enough*** for many APL primitives.
- Simplify certain primitives into other constructs…
- Multiple backends...

APL	*op*(*s*)		TySc(*op*)
	addi, …	:	$\texttt{int} \to \texttt{int} \to \texttt{int}$
	addd, …	:	$\texttt{double} \to \texttt{double} \to \texttt{double}$
⍳	iota	:	$\texttt{int} \to [\texttt{int}]^1$
¨	each	:	$\forall\alpha\beta\gamma.(\alpha \to \beta) \to [\alpha]^\gamma \to [\beta]^\gamma$
/	reduce	:	$\forall\alpha\gamma.(\alpha \to \alpha \to \alpha) \to \alpha \to [\alpha]^{1+\gamma} \to [\alpha]^\gamma$
⍴	shape	:	$\forall\alpha\gamma.[\alpha]^\gamma \to \mathbf{Sh}\ \gamma$
⍴	reshape0	:	$\forall\alpha\gamma\gamma'.\mathbf{Sh}\ \gamma' \to [\alpha]^\gamma \to [\alpha]^{\gamma'}$
⍴	reshape	:	$\forall\alpha\gamma\gamma'.\mathbf{Sh}\ \gamma' \to \alpha \to [\alpha]^\gamma \to [\alpha]^{\gamma'}$
⌽	reverse	:	$\forall\alpha\gamma.[\alpha]^\gamma \to [\alpha]^\gamma$
⌽	rotate	:	$\forall\alpha\gamma.\texttt{int} \to [\alpha]^\gamma \to [\alpha]^\gamma$
⍉	transp	:	$\forall\alpha\gamma.[\alpha]^\gamma \to [\alpha]^\gamma$
⍉	transp2	:	$\forall\alpha\gamma.\mathbf{Sh}\ \gamma \to [\alpha]^\gamma \to [\alpha]^\gamma$
↑	take	:	$\forall\alpha\gamma.\texttt{int} \to \alpha \to [\alpha]^\gamma \to [\alpha]^\gamma$
↓	drop	:	$\forall\alpha\gamma.\texttt{int} \to [\alpha]^\gamma \to [\alpha]^\gamma$
⊃	first	:	$\forall\alpha\gamma.\alpha \to [\alpha]^\gamma \to \alpha$
	zipWith	:	$\forall\alpha_1\alpha_2\beta\gamma.(\alpha_1 \to \alpha_2 \to \beta) \to [\alpha_1]^\gamma \to [\alpha_2]^\gamma \to [\beta]^\gamma$
,	cat	:	$\forall\alpha\gamma.[\alpha]^{\gamma+1} \to [\alpha]^{\gamma+1} \to [\alpha]^{\gamma+1}$
,	cons	:	$\forall\alpha\gamma.[\alpha]^\gamma \to [\alpha]^{\gamma+1} \to [\alpha]^{\gamma+1}$
,	snoc	:	$\forall\alpha\gamma.[\alpha]^{\gamma+1} \to [\alpha]^\gamma \to [\alpha]^{\gamma+1}$

Value typing $\boxed{\Gamma \vdash v : \tau}$

$$\frac{\vdash \delta : \rho}{\Gamma \vdash [\vec{i}]^\delta : [\texttt{int}]^\rho} \ (7) \qquad \frac{\vdash \delta : \rho}{\Gamma \vdash [\vec{r}]^\delta : [\texttt{double}]^\rho} \ (8)$$

$$\frac{}{\Gamma \vdash [\vec{i}]^{\langle n \rangle} : \mathbf{Sh}\ n} \ (9) \qquad \frac{\Gamma, x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda x.e : \tau \to \tau'} \ (10)$$

$$\frac{}{\Gamma \vdash [n]^{\langle 1 \rangle} : \mathbf{VI}\ n} \ (11) \qquad \frac{}{\Gamma \vdash n : \mathbf{I}\ n} \ (12)$$

Expression typing $\boxed{\Gamma \vdash e : \tau}$

$$\frac{\Gamma \vdash e : \mathbf{I}\ n}{\Gamma \vdash [e] : \mathbf{VI}\ n} \ (13) \qquad \frac{\Gamma(x) \geq \tau}{\Gamma \vdash x : \tau} \ (14) \qquad \frac{\tau \subseteq \tau' \quad \Gamma \vdash e : \tau}{\Gamma \vdash e : \tau'} \ (15)$$

$$\frac{\Gamma \vdash e_i : \kappa \quad i = [0; n[}{\Gamma \vdash [\vec{e}^{(n)}] : [\kappa]^1} \ (16) \qquad \frac{\Gamma \vdash e_i : \texttt{int} \quad i = [0; n[}{\Gamma \vdash [\vec{e}^{(n)}] : \mathbf{Sh}\ n} \ (17)$$

$$\frac{\Gamma \vdash e_1 : \tau' \to \tau \quad \Gamma \vdash e_2 : \tau'}{\Gamma \vdash e_1\ e_2 : \tau} \ (18) \qquad \frac{\Gamma \vdash e_1 : \tau \quad \mathrm{fv}(\vec{\alpha}\vec{\gamma}) \cap \mathrm{fv}(\Gamma, \tau') = \emptyset \quad \Gamma, x : \forall\vec{\alpha}\vec{\gamma}.\tau \vdash e_2 : \tau'}{\Gamma \vdash \texttt{let}\ x = e_1\ \texttt{in}\ e_2 : \tau'} \ (19)$$

$$\frac{\mathrm{TySc}(op) \geq \tau_0 \to \ldots \to \tau_{(n-1)} \to \tau \quad \mathrm{arity}(op) = n \quad \Gamma \vdash e_i : \tau_i \quad i = [0; n[}{\Gamma \vdash op(\vec{e}^{(n)}) : \tau} \ (20)$$

Small Step Reductions $\boxed{e \hookrightarrow e' / \mathbf{err}}$

$$\frac{e \hookrightarrow e' \quad E \neq [\cdot]}{E[e] \hookrightarrow E[e']} \ (21) \qquad \frac{e \hookrightarrow \mathbf{err} \quad E \neq [\cdot]}{E[e] \hookrightarrow \mathbf{err}} \ (22)$$

$$\frac{}{\texttt{let}\ x = v\ \texttt{in}\ e \hookrightarrow e[v/x]} \ (23) \qquad \frac{}{(\lambda x.e)\ v \hookrightarrow e[v/x]} \ (24)$$

$$\frac{}{[\vec{a}^{(n)}] \hookrightarrow [\vec{a}^{(n)}]^{\langle n \rangle}} \ (25)$$

$$\frac{i = i_1 + i_2}{\texttt{addi}(i_1, i_2) \hookrightarrow i} \ (26) \qquad \frac{d = d_1 + d_2}{\texttt{addd}(d_1, d_2) \hookrightarrow d} \ (27)$$

$$\frac{n \geq 0}{\texttt{iota}(n) \hookrightarrow [1, \cdots, n]^{\langle n \rangle}} \ (28) \qquad \frac{n < 0}{\texttt{iota}(n) \hookrightarrow \mathbf{err}} \ (29)$$

$$\frac{e = [v_f\ a_0, \cdots, v_f\ a_{(n-1)}]}{\texttt{each}(v_f, [\vec{a}^{(n)}]^\delta) \hookrightarrow \texttt{reshape0}(\delta, e)} \ (30)$$

$$\frac{\delta = \langle \vec{n}, m \rangle \quad k = \mathrm{product}(\vec{n}) \quad i = [0; m[\quad e_i = v_f\ a_{(i*k)}\ (\cdots (v_f\ a_{(i*k+m-1)}\ v) \cdots)}{\texttt{reduce}(v_f, v, [\vec{a}^{(n)}]^\delta) \hookrightarrow \texttt{reshape0}(\langle \vec{n} \rangle, [\vec{e}^{(k)}])} \ (31)$$

$$\frac{m = \mathrm{product}(\delta') \quad f(i) = i \bmod n \quad n > 0}{\texttt{reshape}(\delta', a, [\vec{a}^{(n)}]^\delta) \hookrightarrow [a_{f(0)}, \cdots, a_{f(m-1)}]^{\delta'}} \ (32)$$

$$\frac{m = \mathrm{product}(\delta') \quad a_i = a \quad i = [0; m[}{\texttt{reshape}(\delta', a, []^\delta) \hookrightarrow [a_0, \cdots, a_{(m-1)}]^{\delta'}} \ (33)$$

$$\frac{\delta' = \mathrm{rev}(\delta) \quad f = \mathrm{fromSh}_{\delta'}\ o\ \mathrm{rev}\ o\ \mathrm{toSh}_\delta}{\texttt{transp}([\vec{a}^{(n)}]^\delta) \hookrightarrow [a_{f(0)}, \cdots, a_{f(n-1)}]^{\delta'}} \ (34)$$

TAIL Example

APL → TAIL → Futhark

APL:

```
mean ← +/÷≢
var ← mean({ω*2}⊢-mean)
stddev ← {ω*0.5} var
all ← mean, var, stddev
⎕ ← all 54 44 47 53 51 48 52 53 52 49 48
```

TAIL:

```
let v2:[int]1 = [54,44,47,53,51,48,52,53,52,49,48,52] in
let v1:[int]0 = 11 in
let v15:[double]1 = each(fn v14:[double]0 =>
subd(v14,divd(i2d(reduce(addi,0,v2)),i2d(v1))),each(i2d,v2)) in
let v17:[double]1 = each(fn v16:[double]0 => powd(v16,2.0),v15) in
let v21:[double]0 = divd(reduce(addd,0.0,v17),i2d(v1)) in
let v31:[double]1 = each(fn v30:[double]0 =>
subd(v30,divd(i2d(reduce(addi,0,v2)),i2d(v1))),each(i2d,v2)) in
let v33:[double]1 = each(fn v32:[double]0 => powd(v32,2.0),v31) in
let v41:[double]1 =
prArrD(cons(divd(i2d(reduce(addi,0,v2)),i2d(v1)),[divd(reduce(add
d,0.0,v33),i2d(v1)),powd(v21,0.5)])) in 0
```

Type check: Ok

Evaluation:

[3](50.0909,8.8099,2.9681)

Simple interpreter

Compiling Primitives

APL → **TAIL** → Futhark

APL:

Guibas and Wyatt, POPL'78

```
dot ← {
  WA ← (1↓ρω),ρα
  KA ← (⊃ρρα)-1
  VA ← ⍳ ⊃ ρWA
  ZA ← (KA⌽¯1↓VA),¯1↑VA
  TA ← ZA⍉WAρα
  WB ← (¯1↓ρα),ρω
  KB ← ⊃ ρρα
  VB ← ⍳ ⊃ ρWB
  ZB0 ← (-KB) ↓ KB ⌽ ⍳(⊃ρVB)
  ZB ← (¯1↓(⍳ KB)),ZB0,KB
  TB ← ZB⍉WBρω
  αα / TA ωω TB
}

A ← 3 2 ρ ⍳ 5
B ← ⍉ A
R ← A + dot × B
R2 ← ×/ +/ R
```

TAIL:

```
let v1:[int]2 = reshape([3,2],iotaV(5)) in
let v2:[int]2 = transp(v1) in
let v9:[int]3 = transp2([2,1,3],reshape([3,3,2],v1)) in
let v15:[int]3 = transp2([1,3,2],reshape([3,2,3],v2)) in
let v20:[int]2 = reduce(addi,0,zipWith(muli,v9,v15)) in
let v25:[int]0 = reduce(muli,1,reduce(addi,0,v20)) in
i2d(v25)
```

Evaluating
Result is [](65780.0)

Notice: Quite a few simplifications happen at TAIL level..

Futhark

Pure eager **functional language** with second-order parallel array constructs.

Support for “imperative-like” language constructs for iterative computations (i.e., graph shortest path).

A ***sequentialising*** compiler...

Close to performance obtained with hand-written OpenCL GPU code.

```
let addTwo (a:[]i32) : []i32 = map (+2) a
let sum (a:[]i32) : i32 = reduce (+) 0 a
let sumrows(a:[][]i32) : []i32 = map sum a
let main(n:i32) : i32 =
  loop x=1 for i < n do x * (i+1)
```

Performs general optimisations

- *Constant folding*. E.g., remove branch inside code for take(n,a) if n ≤ ⊃ρa.
- *Loop fusion*. E.g., fuse the many small “vectorised” loops in idiomatic APL code.

Attempts at flattening nested parallelism

- E.g., reduction (/) inside each (¨).

Allows for indexing and sequential loops

- Needed for indirect indexing and ⍣.

Performs low-level GPU optimisations

- E.g., optimise for coalesced memory accesses.

An Example

APL → TAIL → Futhark

APL:

```
f ← { 2 ÷ ⍵ + 2 }                ⍝ Function \x. 2 / (x+2)
X ← 1000000                      ⍝ Valuation points per unit
domain ← 10 × (⍳X) ÷ X           ⍝ Integrate from 0 to 10
integral ← +/ (f¨domain)÷X       ⍝ Compute integral
```

TAIL:

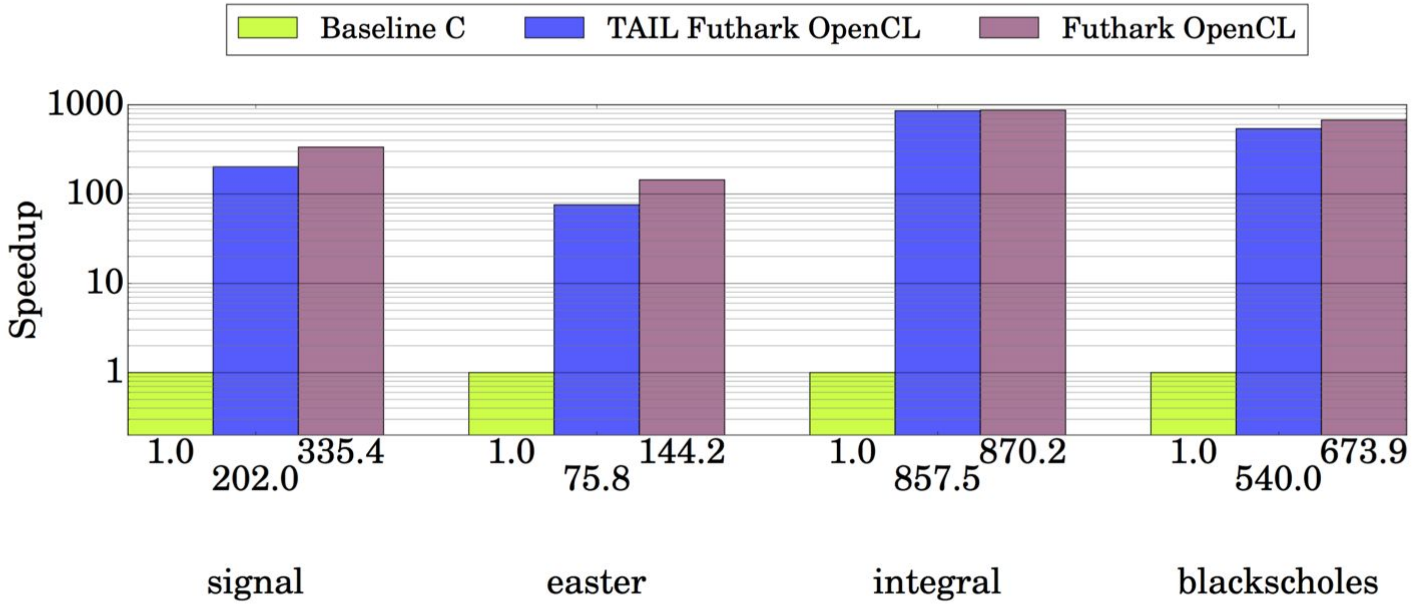
```
let domain:<double>1000000 =
  eachV(fn v4:[double]0 => muld(10.0,v4),
   eachV(fn v3:[double]0 => divd(v3,1000000.0),
    eachV(i2d,iotaV(1000000)))) in
let integral:[double]0 =
  reduce(addd,0.0,
   eachV(fn v9:[double]0 => divd(v9,1000000.0),
    eachV(fn v7:[double]0 => divd(2.0,addd(v7,2.0)),
     domain))) in
integral
```

Futhark - before optimisation:

```
let domain =
 map (\ (t_v4: f64): f64 -> 10.0f64*t_v4)
  (map (\ (t_v3: f64): f64 -> t_v3/1000000.0f64)
   (map i2d (map (\ (x: int): int -> x+1)
    (iota 1000000))))
let integral =
 reduce (+) 0.0f64
  (map (\ (t_v9: f64): f64 -> t_v9/1000000.0f64)
   (map (\ (t_v7: f64): f64 -> 2.0f64/(t_v7+2.0f64))
    domain))
In integral
```

Notice: TAIL2Futhark compiler is quite straightforward...

Performance Compute-bound Examples

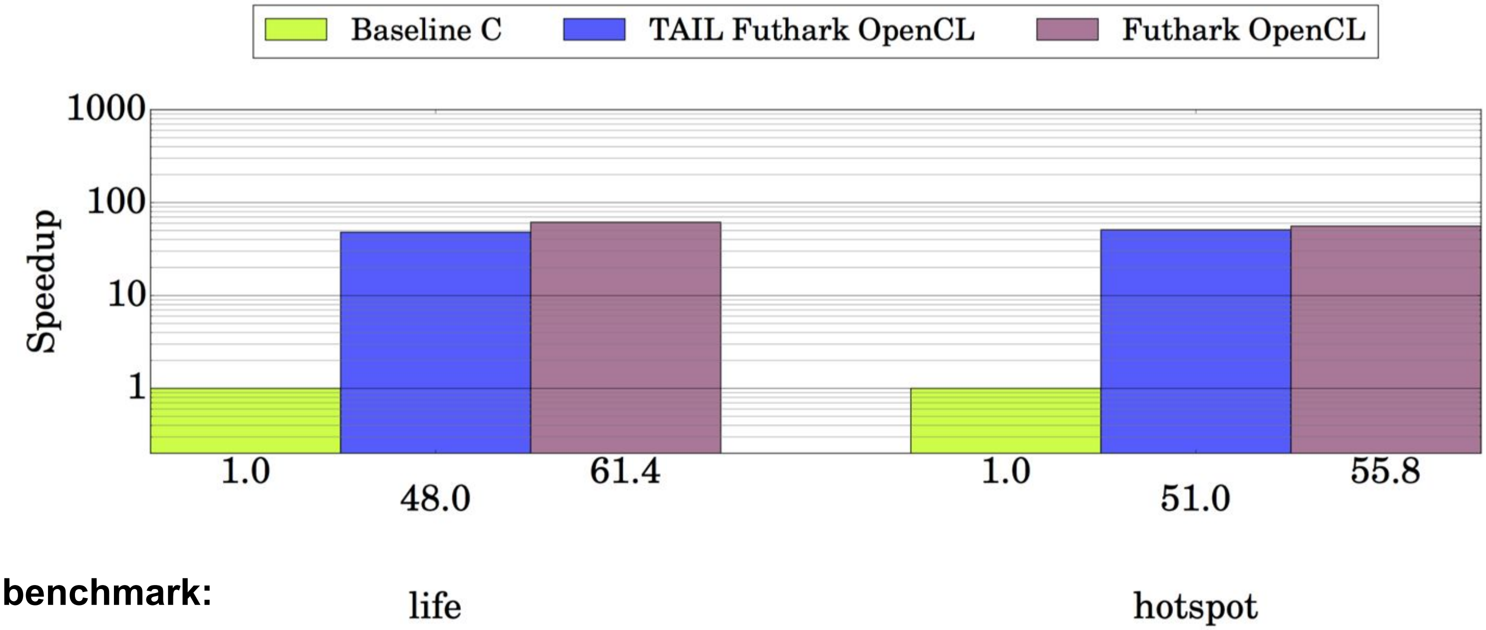


Integral benchmark:

```
f ← { 2 ÷ ω + 2 }                 ⍝ Function \x. 2 / (x+2)
X ← 10000000                      ⍝ Valuation points per unit
domain ← 10 × (⍳X) ÷ X            ⍝ Integrate from 0 to 10
integral ← +/ (f¨domain)÷X        ⍝ Compute integral
```

OpenCL runtimes from an NVIDIA GTX 780
CPU runtimes from a Xeon E5-2650 @ 2.6GHz

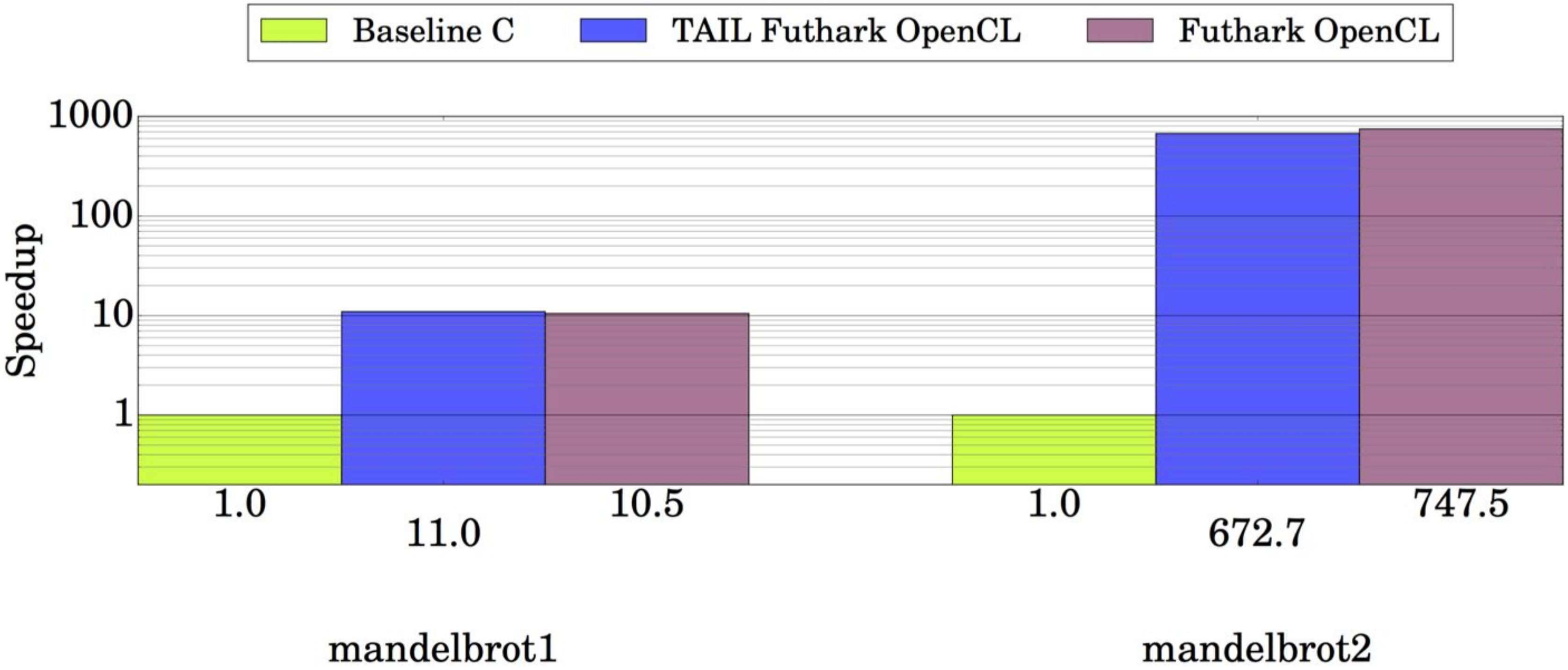
Performance Stencils



Life benchmark:

```
life ← {
  rs ← { (¯1⌽ω) + ω + 1⌽ω }
  n ← (rs ¯1⊖ω) + (rs ω) + rs 1⊖ω
  (n=3) ∨ (n=4) ∧ ω
}
res ← +/+/ (life ⍣ 100) board
```

Performance Mandelbrot



New Features Since Dyalog'16

Complex number support:

```
⍝ Mandelbrot one-liner:
⍝ Compared to Dyalog APL, additional parentheses are needed around
⍝ bindings (←) and around the power operator (⍣).

⎕ ← ' #'[1+9>|({m+ω×ω}⍣9)(m←¯3×.7j.5-⍉a∘.+0j1×(a←(1+⍳n+1)÷(n←28)))]
```

Efficient parallel segmented reductions (Troels' + Rasmus' FHPC'17 paper).

- A special segmented reduction form is possible in APL:

+/20000 10⍴⍳200000 **+/100 2000⍴⍳200000**

Futhark components (library routines).

- Linear algebra routines, sobol sequences, sorting, random numbers, ...

Many Futhark internal optimisations.

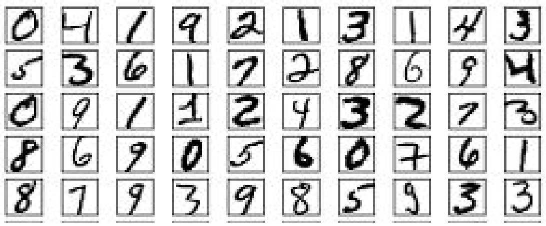
Neural Network for Digit Recognition

Task: Train a 3-layer neural network using back-propagation.

MNIST data set:

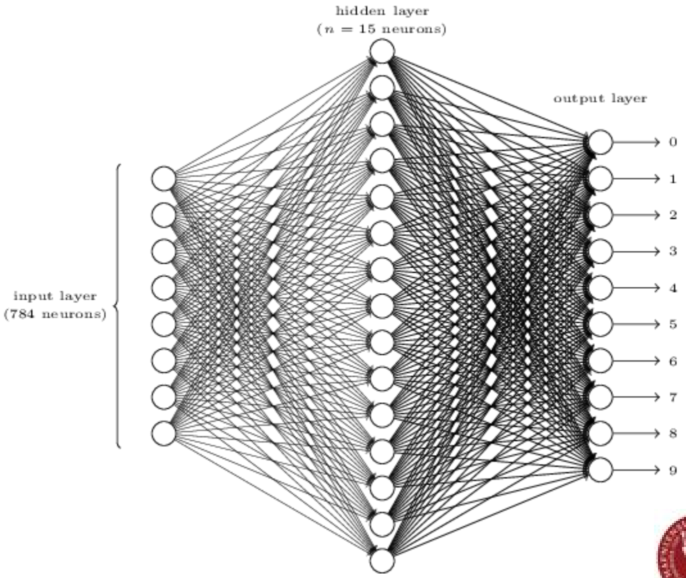
- *Training* set size: 50,000 classified images
  (28x28 pixel intensities; floats)

- *Test* set size: 10,000 classified images



Network:

Input layer	*Layer 2 (Hidden)*	*Layer 3 (Output)*
784 (28x28)	30 sigmoid neurons	10 sigmoid neurons
	weights: 30x784 matrix **biases**: 30 vector	**weights**: 10x30 matrix **biases**: 10 vector



Some APL NN Snippets

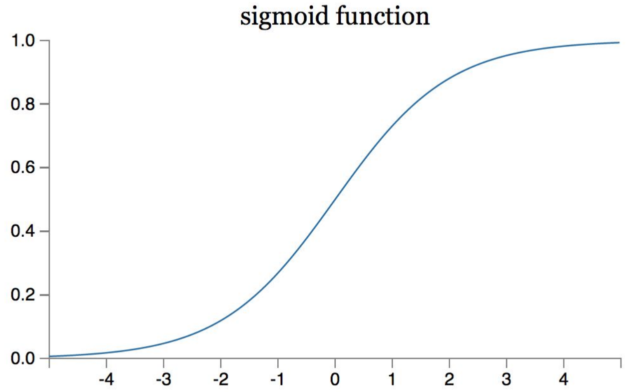
```
⍝ The sigmoid function
sigmoid ← { ÷1+*-ω }
```

$$\sigma(z) \equiv \frac{1}{1+e^{-z}}$$

```
⍝ Turn a digit into a 10d unit vector
from_digit ← { ω=¯1+ι10 }

⍝ Predict a digit based on the output
⍝ layer's activation vector
predict_digit ← { ¯1++/(ι≢ω)×ω=⌈/ω }

⍝ Apply a 3-layer network to an input vector
feedforward3 ← {
  feedforward ← {
    b ← α[1] ⋄ w ← α[2]
    sigmoid b + w +.× ω
  }
  α[2] feedforward (α[1] feedforward ω)
}
```



0	0.23
1	0.32
2	0.03
3	0.45
:	:
9	1.25

→ 9

NN Implementation in Futhark

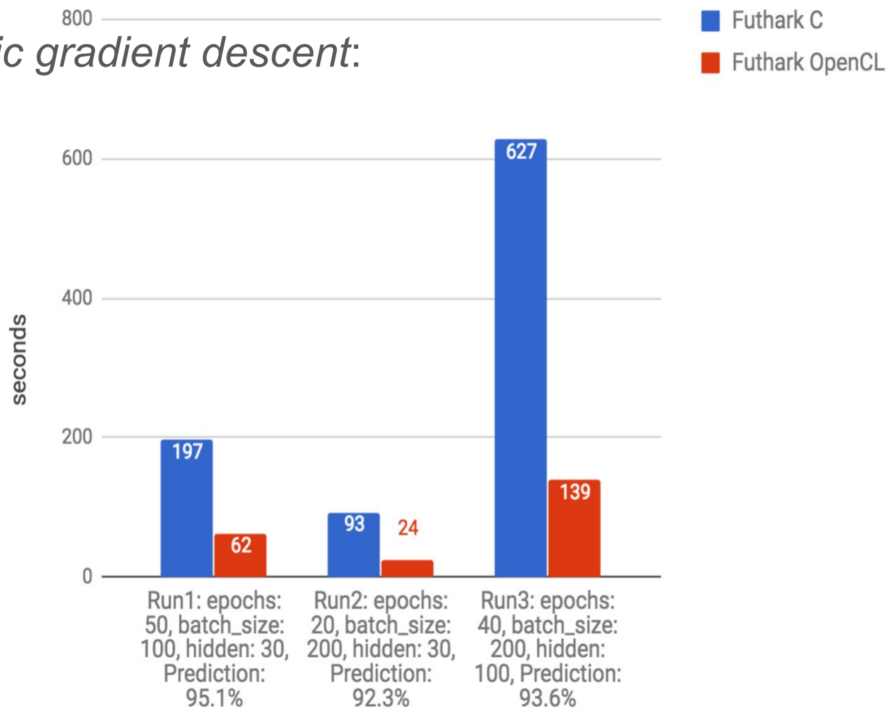
Original in Python - neuralnetworksanddeeplearning.com

Back-propagation algorithm based on *stochastic gradient descent*:

```
⍝ Pseudo code:
epochs ← 20
N ← ({ train_data ← random_permute train_data
       batches ← split train_data
       nablas ← ω backprop batches
       ω − +/nablas
     }⍣epochs) init_N (28×28) 30 10
```

Futhark supports arrays of ‘pairs of arrays’, which can be processed in parallel using the generic Futhark **map** function.

The argument function to **map** may itself return structured values.



NN Implementation in APL

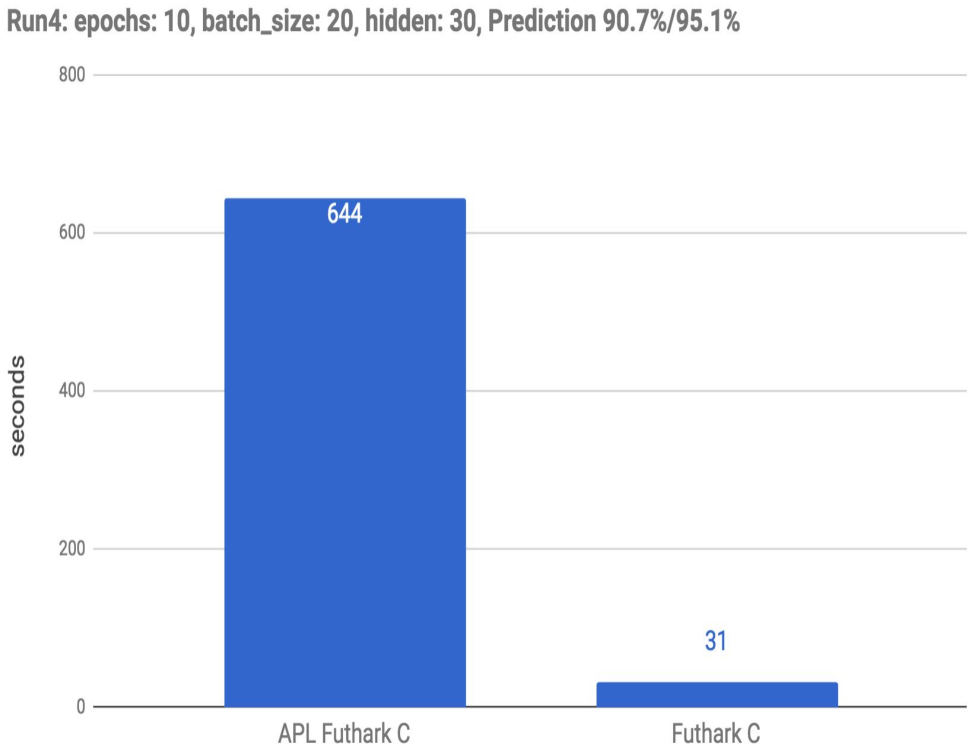
20x slowdown with respect to native Futhark.

More investigations are needed to identify the performance issues.

400 lines of APL code.

How does Dyalog APL perform on this benchmark?

How should it be written in Dyalog APL for it to hit peak performance?



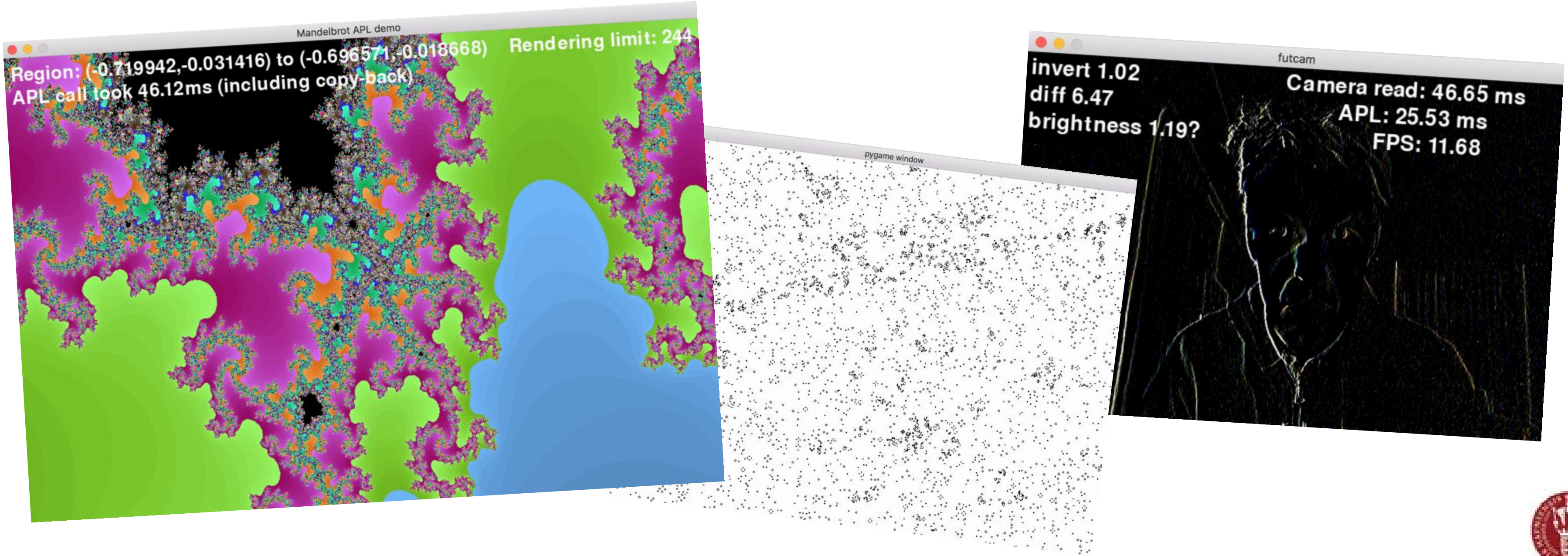
See https://github.com/melsman/neural-networks-and-deep-learning

Interoperability Demos

Mandelbrot, Life, AplCam

With Futhark, we can generate reusable *modules* in various languages (e.g, Python) that internally execute on the GPU using OpenCL.

```
onChannels ← {
  m ← 3 1 2 ⍉ ω
  m ← (αα h w ρm) ⍪ (αα h w ρ1↓m) ⍪ αα h w ρ2↓m
  2 3 1 ⍉ 3 h w ρ m
}
diff ← {
  n ← ⌈degree
  255⌊n×+ω−1⌽ω
}
image ← diff onChannels image
```



Related Work

APL Compilers

- Co-dfns compiler by Aaron Hsu. Papers in ARRAY'14 and ARRAY'16.
- C. Grelck and S.B. Scholz. *Accelerating APL programs with SAC*. APL'99.
- R. Bernecky. *APEX: The APL parallel executor*. MSc Thesis. University of Toronto. 1997.
- L.J. Guibas and D.K. Wyatt. *Compilation and delayed evaluation in APL*. POPL'78.

Type Systems for APL like Languages

- K. Trojahner and C. Grelck. *Dependently typed array programs don't go wrong*. NWPT'07.
- J. Slepak, O. Shivers, and P. Manolios. *An array-oriented language with static rank polymorphism*. ESOP'14.

Futhark work

- Papers on language and optimisations available from hiperfit.dk.
- Futhark available from futhark-lang.org.

Other functional languages for GPUs

- Accelerate. Haskell library/embedded DSL.
- Obsidian. Haskell embedded DSL.
- FCL. Low-level functional GPU programming. FHPC'16.

Libraries for GPU Execution

- Thrust, cuBLAS, cuSPARSE, ...

Conclusions

We have managed to get a (small) subset of APL to run efficiently on GPUs.

- https://github.com/HIPERFIT/futhark-fhpc16
- https://github.com/henrikurms/tail2futhark
- https://github.com/melsman/apltail

Future Work

- More real-world benchmarks.
- Support a wider subset of APL.
- Improve interoperability...
- Add support for APL “type annotations” for specifying programmer intentions...

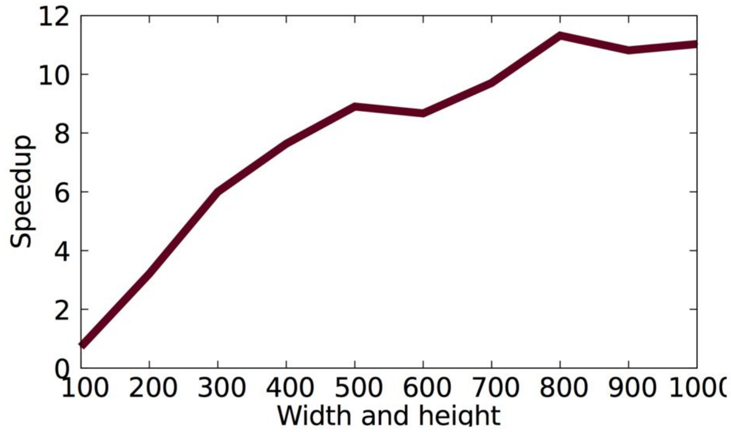
HIPERFIT

Different Mandelbrot Implementations

<u>Parallel inner loop:</u>
mandelbrot1.apl

```
seq for i < depth:
  par for j < n:
    points[j] = f(points[j])
```

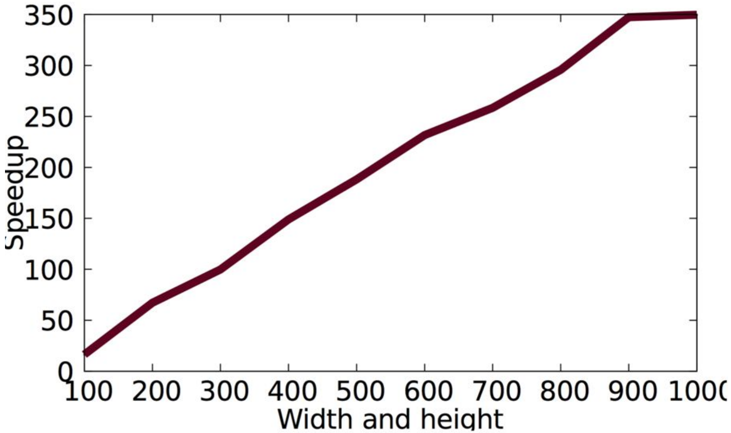
Memory bound



<u>Parallel outer loop:</u>
mandelbrot2.apl

```
par for j < n:
  p = points[j]
  seq for i < depth:
    p = f(p)
  points[j] = p
```

Compute bound



mandelbrot1.apl and mandelbrot2.apl

```
⍝ grid-size in left argument (e.g., (1024 768))
⍝ X-range, Y-range in right argument

mandelbrot1 ← {
  X ← ⊃⍺ ⋄ Y ← ⊃1↓⍺
  xRng ← 2↑⍵ ⋄ yRng ← 2↓⍵
  dx ← ((xRng[2])-xRng[1]) ÷ X
  dy ← ((yRng[2])-yRng[1]) ÷ Y
  cx ← Y X ⍴ (xRng[1]) + dx × ⍳X    ⍝ real plane
  cy ← ⍉ X Y ⍴ (yRng[1]) + dy × ⍳Y   ⍝ img plane
  mandel1 ← {                        ⍝ one iteration
    zx ← Y X ⍴⍵[1] ⋄ zy ← Y X ⍴⍵[2]
    count ← Y X ⍴ ⍵[3]               ⍝ count plane
    zzx ← cx + (zx × zx) - zy × zy
    zzy ← cy + (zx × zy) + zx × zy
    conv ← 4 > (zzx × zzx) + zzy × zzy
    count2 ← count + 1 - conv
    (zzx zzy count2)
  }
  pl ← Y X ⍴ 0                       ⍝ zero-plane
  N ← 255                            ⍝ iterations
  res ← (mandel1 ⍣ N) (pl pl pl)
  res[3] ÷ N                         ⍝ count plane
}
```

```
mandelbrot2 ← {
  X ← ⊃⍺ ⋄ Y ← ⊃1↓⍺
  xRng ← 2↑⍵ ⋄ yRng ← 2↓⍵
  dx ← ((xRng[2])-xRng[1]) ÷ X
  dy ← ((yRng[2])-yRng[1]) ÷ Y
  cxA ← Y X ⍴ (xRng[1])+dx×⍳X    ⍝ real plane
  cyA ← ⍉ X Y ⍴ (yRng[1])+dy×⍳Y  ⍝ img plane
  N ← 255                        ⍝ iterations
  mandel1 ← {
    cx ← ⍺ ⋄ cy ← ⍵
    f ← {
      arg ← ⍵
      x ← arg[1] ⋄ y ← arg[2]
      count ← arg[3]
      dummy ← arg[4]
      zx ← cx+(x×x)-(y×y)
      zy ← cy+(x×y)+(x×y)
      conv ← 4 > (zx × zx) + zy × zy
      count2 ← count + 1 - conv
      (zx zy count2 dummy)
    }
    res ← (f⍣N) (0 0 0 'dummy')  ⍝ N iterations
    res[3]
  }
  res ← cxA mandel1¨ cyA
  res ÷ N
}
```