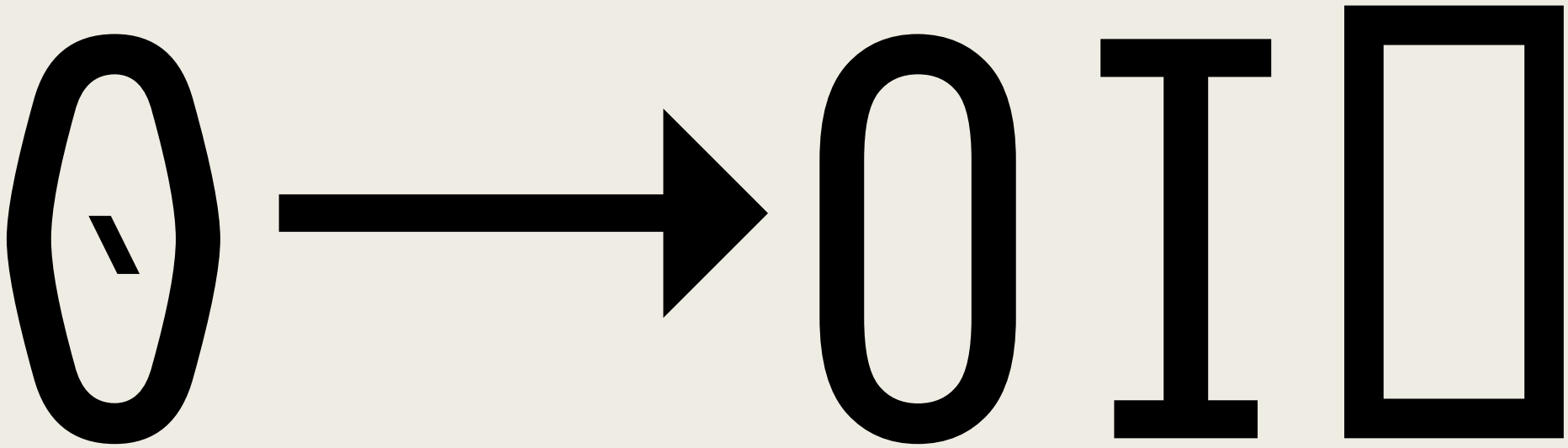
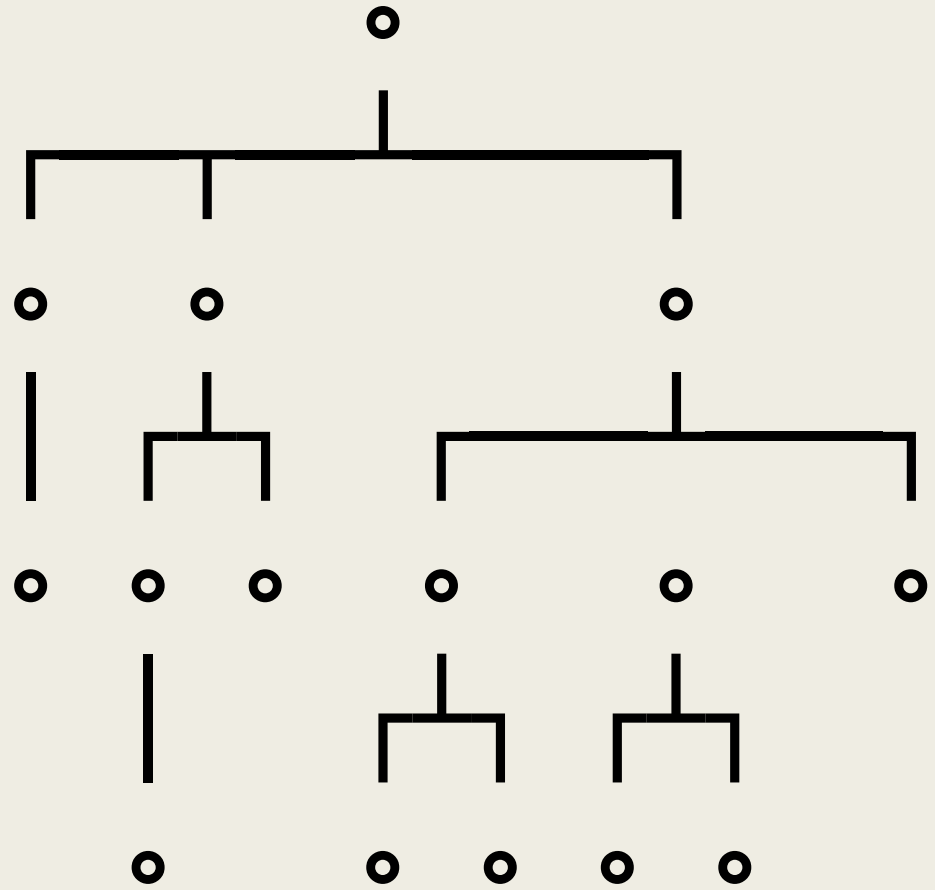
TREE WRANGLING THE APL WAY

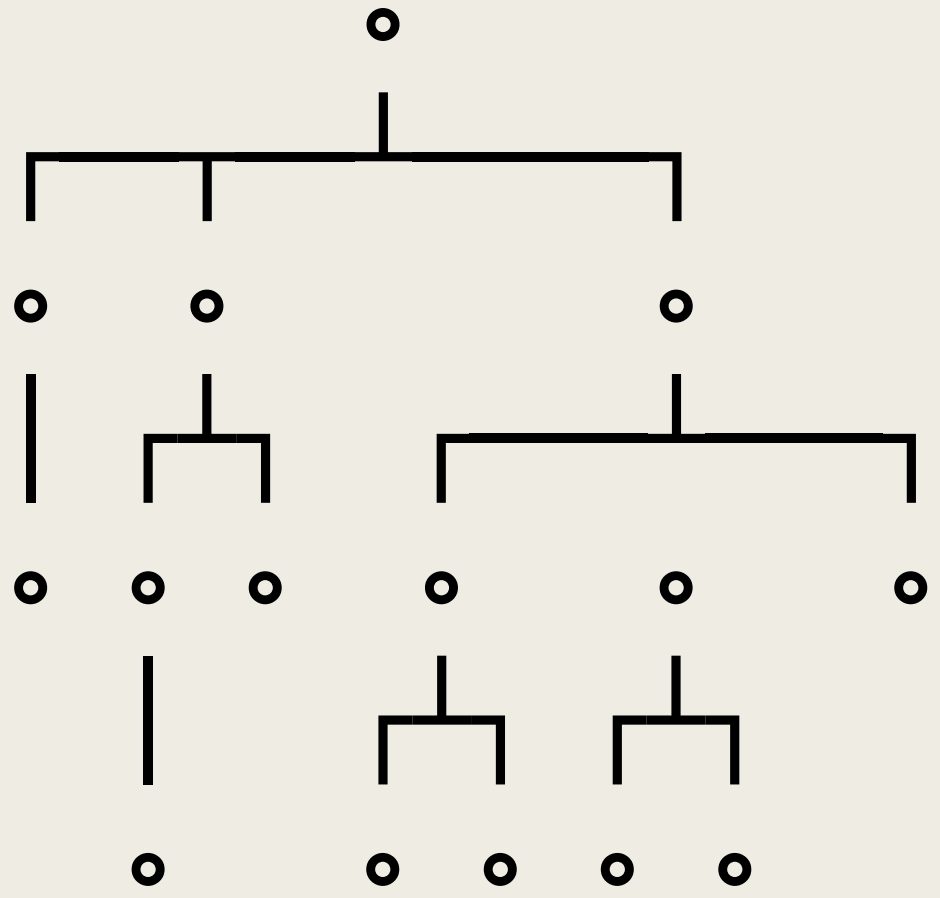
Aaron W. Hsu, Indiana University

Dyalog ‘18, Belfast





Depth:



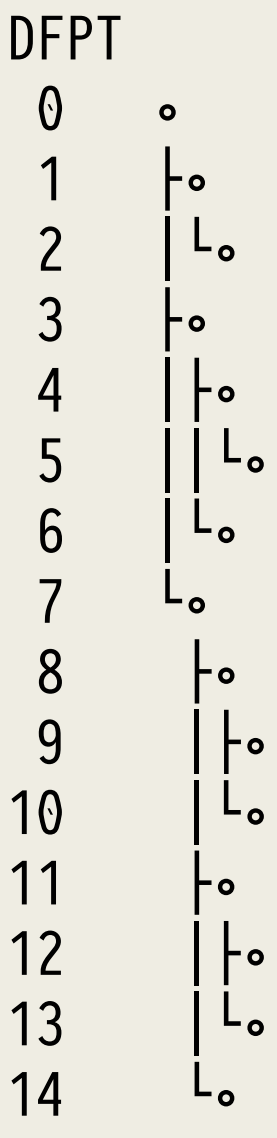
0

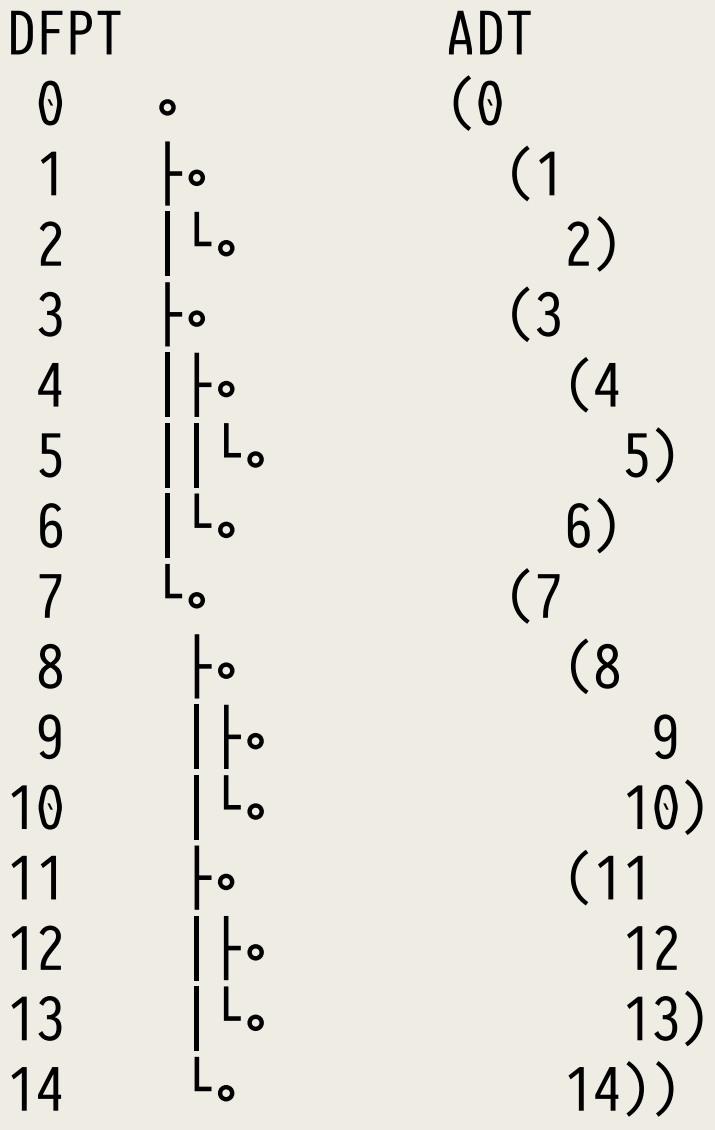
1

2

3


I DON'T USUALLY USE ONE SLIDE,
BUT WHEN I DO...
memegenerator.net


DFPT
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14


DFPT
ADT
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
(0
(1
2)
(3
(4
5)
6)
(7
(8
9
10)
(11
12
13)
14))

```
DFPT     Depth  ADT
 0    ∘     0    (0
 1    ├∘    1      (1
 2    │└∘   2        2)
 3    ├∘    1      (3
 4    │├∘   2        (4
 5    ││└∘  3          5)
 6    │└∘   2        6)
 7    └∘    1      (7
 8     ├∘   2        (8
 9     │├∘  3          9
10     │└∘  3          10)
11     ├∘   2        (11
12     │├∘  3          12
13     │└∘  3          13)
14     └∘   2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1
 2   │└∘  2        2)
 3   ├∘   1      (3
 4   │├∘  2        (4
 5   ││└∘ 3          5)
 6   │└∘  2        6)
 7   └∘   1      (7
 8    ├∘  2        (8
 9    │├∘ 3          9
10    │└∘ 3          10)
11    ├∘  2        (11
12    │├∘ 3          12
13    │└∘ 3          13)
14    └∘  2        14))
```

```
DFPT    Depth  ADT        APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0         adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1       ids←∈adt ⍤ ⍳15
 2   │└∘  2        2)
 3   ├∘   1      (3
 4   │├∘  2        (4
 5   ││└∘ 3          5)
 6   │└∘  2        6)
 7   └∘   1      (7
 8    ├∘  2        (8
 9    │├∘ 3           9
10    │└∘ 3          10)
11    ├∘  2        (11
12    │├∘ 3          12
13    │└∘ 3          13)
14    └∘  2        14))
```

```
DFPT    Depth  ADT        APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0         adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1       ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)     d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4
 5   ││└∘ 3          5)
 6   │└∘  2        6)
 7   └∘   1      (7
 8    ├∘  2        (8
 9    │├∘ 3           9
10    │└∘ 3          10)
11    ├∘  2        (11
12    │├∘ 3          12
13    │└∘ 3          13)
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0
 6   │└∘  2        6)       1     3           7
 7   └∘   1      (7             2     4     6     8        11        14
 8    ├∘  2        (8                   5           9 10    12 13
 9    │├∘ 3          9
10    │└∘ 3          10)
11    ├∘  2        (11
12    │├∘ 3          12
13    │└∘ 3          13)
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)       1     3           7
 7   └∘   1      (7             2     4     6     8        11        14
 8    ├∘  2        (8                   5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10)
11    ├∘  2        (11
12    │├∘ 3          12
13    │└∘ 3          13)
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)       1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7             2     4     6     8        11        14
 8    ├∘  2        (8                   5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10)
11    ├∘  2        (11
12    │├∘ 3          12
13    │└∘ 3          13)
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)       1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7             2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                   5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10)
11    ├∘  2        (11
12    │├∘ 3          12
13    │└∘ 3          13)
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11      0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←
13    │└∘ 3          13) l←
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0
13    │└∘ 3          13) l←0
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0
13    │└∘ 3          13) l←0  1
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1
13    │└∘ 3          13) l←0  1  2
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍀ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11      0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0
13    │└∘ 3          13) l←0  1  2  1
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍤ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3
13    │└∘ 3          13) l←0  1  2  1  4
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4
13    │└∘ 3          13) l←0  1  2  1  4  5
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍤ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3
13    │└∘ 3          13) l←0  1  2  1  4  5  4
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍀ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0  7
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3  8
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11      0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0  7  8
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3  8  9
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0  7  8  8
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3  8  9  9
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                  5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0  7  8  8  7
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3  8  9  9  8
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0  7  8  8  7 11
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3  8  9  9  8 12
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍤ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)       1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7           2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                    5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11      0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0  7  8  8  7 11 11
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3  8  9  9  8 12 12
14    └∘  2        14))
```

```
DFPT    Depth  ADT       APL (c.f. ⎕XML/⎕JSON)
 0   ∘    0    (0        adt←(0 (1 2) (3 (4 5) 6) (7 (8 9 10) (11 12 13) 14))
 1   ├∘   1      (1      ids←∈adt ⍳ ⍳15
 2   │└∘  2        2)    d←0 1 2 1 2 3 2 1 2 3 3 2 3 3 2
 3   ├∘   1      (3
 4   │├∘  2        (4    PATH MATRIX (c.f. Hsu, ARRAY 2016, extended)
 5   ││└∘ 3          5)  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 6   │└∘  2        6)      1  1  3  3  3  3  7  7  7  7  7  7  7  7
 7   └∘   1      (7          2     4  4  6     8  8  8 11 11 11 14
 8    ├∘  2        (8                5           9 10    12 13
 9    │├∘ 3           9
10    │└∘ 3           10) PARENT/SIBLING
11    ├∘  2        (11     0  1  2  3  4  5  6  7  8  9 10 11 12 13 14
12    │├∘ 3          12  p←0  0  1  0  3  4  3  0  7  8  8  7 11 11  7
13    │└∘ 3          13) l←0  1  2  1  4  5  4  3  8  9  9  8 12 12 11
14    └∘  2        14))
```

Adding: ,←

Adding: ,←

Deleting: (⍳M)(⊢-1+⍳)(~M)≠P

```
Adding:      ,←
Deleting:    (⍳M)(⊢-1+⍳)(~M)≠P
Updating:    @ []←
```

```
Adding:      ,←
Deleting:    (⍳M)(⊢-1+⍳)(~M)≠P
Updating:    @ []←
Traversing: I@{}⍤≡
```

```
Adding:      ,←
Deleting:    (⍳M)(⊢-1+⍳)(~M)≠P
Updating:    @ []←
Traversing: I@{}⍤≡
Selection:   = ∈ ⍳ ⍳
```

```
⍝ Depth of a Tree
d⊣{z⊣d+←ω≠z←α[ω]}⍣≡⍨p⊣d←p≠⍳≢p
```

ꟊ Binding Table

```
bv←I@{1=t[ω]}⍤≡⍨i@(p[i←⍳1=t[p]])ι≠p
```

```
⍝ Lift Functions
i←⊢⍳(t=3)∧p≠⍳s←⍴p
l←i(s+⍳)@{⍵∈i}l ⋄ p l(⊣,I)←⊂i
t k,←10 1⍴¨~¨⍴i
n,←i
p[i]←i
l[j]←⊃(⌽i),j←⊢⍳(p=⍳≠p)∧l=⍳≠l
l[i]←(⍴i)↑(⊃i),i
```

```
⍝ Wrap Return Expressions
i←⍸(t∊0 2)∨(t=1)∧k=0
i(⍸∧)←(t[p]∊3 4)∧~(⍳≠⍳)∊¯1@{⍵=⍳≠⍵}⍳
p,←p[i] ⋄ p[i]←(≠⍳)+⍳≠i
l←i((≠⍳)+⍳)@{⍵∊i}⍳
l,←l[i] ⋄ l[i]←i
t k n,←2 0 0⍴⍨¨≠i
```

Bug-zone!

```
⍘ Wrap Return Expressions
i←(t∈0 2)∨(t=1)∧k=0
i(⍳⍲)←(t[p]∈3 4)∧~(⍳≠1)∈¯1@{⍵=⍳≠⍵}1
```

```
ꟼ Wrap Return Expressions
i←(t∈0 2)∨(t=1)∧k=0
i(⍳⍲)←(t[p]∈3 4)∧~(⍳≠1)∈¯1@{⍵=⍳≠⍵}1
```

$$\{x \diamond y\}$$

⍘ Wrap Return Expressions

```
i←(t∈0 2)∨(t=1)∧k=0
i∧←(t[p]∈3 4)∧~(⍳≠1)∈¯1@{⍵=⍳≠⍵}1
i(⍳∨)←(t∈0 2)∧t[p]=3
```

{x ◊ y}

What’s the Takeaway?

Dear CS PL Community:

We’ve had a good run for 50+ years, it’s time to admit defeat and use APL.

Yours truly,
arcfide

Dear APL Community:

Your time is come again,
 trees are now your oyster.
Don’t gloat too much.

Yours truly,
arcfide

Thank You.