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Dyalog '21

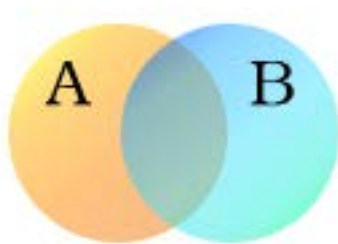
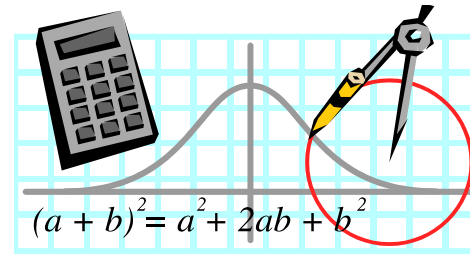
November 8, 2021

Extending the Domain of the Probability Operator in TamStat



Statistics deals primarily with four types of functions:

- Summary Functions
 - Descriptive Statistics
- Probability Distributions
 - Theoretical Models
- Relations
- Logic



Logical Functions – Domain $\{0,1\}$ Range $\{0,1\}$

Logical Functions

- Not (Monadic \sim)
- And ($A \wedge B$)
- Or ($A \vee B$)
- Nand ($A \tilde{\wedge} B$)
- Nor ($A \tilde{\vee} B$)

Relational Functions Applied to Booleans

- IFF (If and only if) ($A=B$)
- Exclusive Or ($A \neq B$)
- Relative Complement ($A < B$)
- Without ($A > B$)
- Implies ($A \leq B$)
- If ($A \geq B$)



Current Probability Operator:

What is the probability that you get at least 3 heads in seven fair coin tosses?

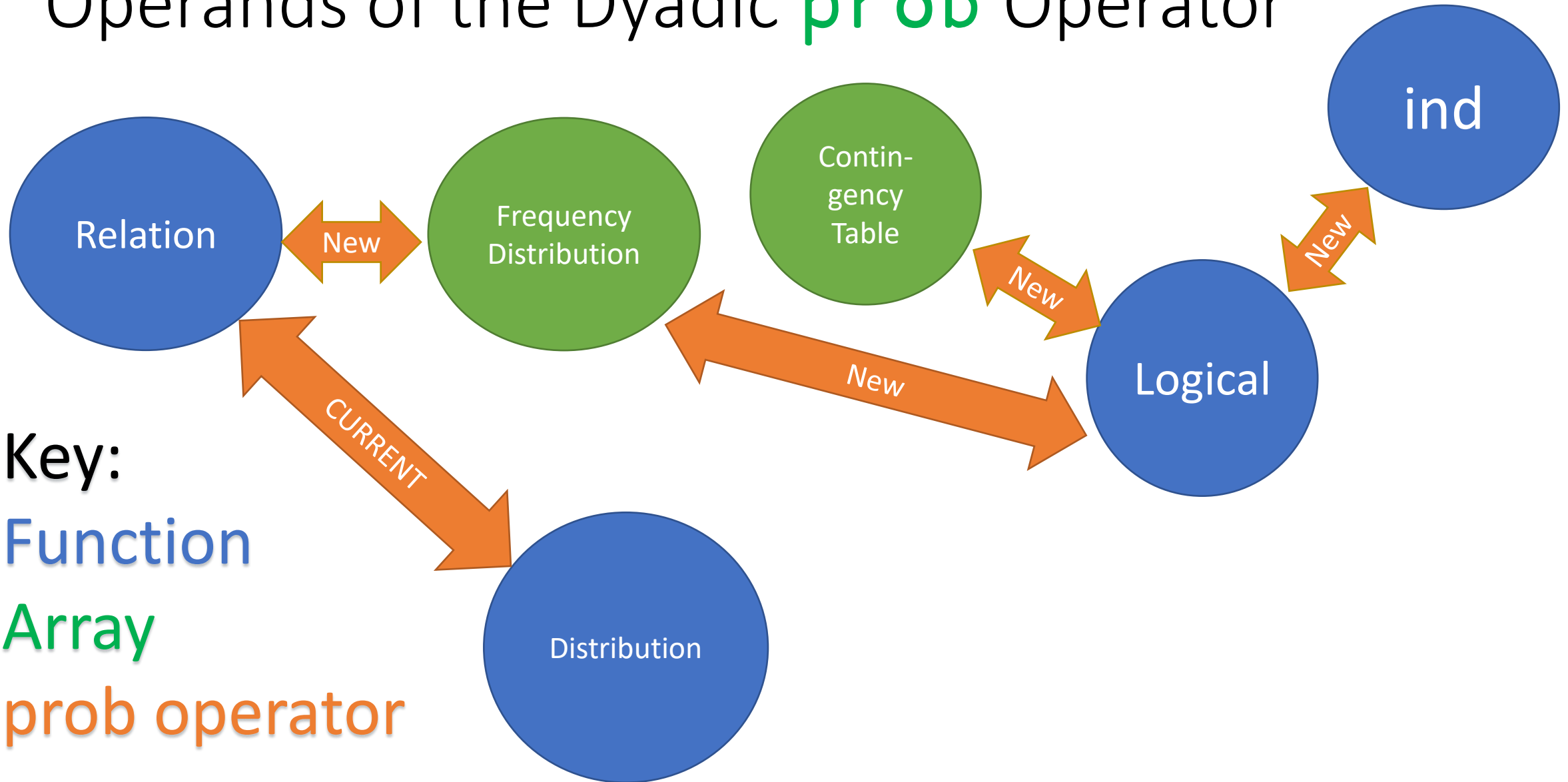
Excel: `=1-BINOM.DIST(2,7,0.5,1)`

R: `pbinom(2,7,0.5,lower.tail=FALSE)`

APL/TamStat:

```
P ← 7 0.5 binomial      prob      ≥      3
-----
      ↓      ↓      ↓      ↓      ↓
    Left  Left  Operator  Right  Right
    Arg  Operand
      P
0.7734375
```

Operands of the Dyadic **prob** Operator



Key:

Function

Array

prob operator

Custom Probabilities Using a Frequency Distribution as Left Operand

Custom←frequency Family

0 2

1 17

2 11

3 7

4 1

Custom prob = 1

0.44737

A Cumulative prob

Custom prob ≤ 1

0.5

A Upper Tail prob

Custom prob > 2

0.21053

A Interval probability

Custom prob between 1 4

0.47368

Extended Syntax of Probability Operator

- The current syntax of the probability operator is:

Result ← [Parameters] (*distribution/Table* **prob** *relation*) Value

- The extended syntax of the probability operator is:

Result ← [Event1] (*logicalFn* **prob** *Table*) Event2

Result ← [Event1] (*logicalFn* **prob** *Variable1* [Variable2]) Event2

Result ← [Prob1] (*logicalFn* **prob** **ind**) Prob2

- *Table* is a matrix containing integers ≥ 0 or probabilities between 0 and 1 and whose first column or first row and column are event names.
- *Variable* is a nested vector or character matrix.
- In all cases $0 \leq \text{Result} \leq 1$.

Rules of Probability (Summary)

Term	Symbol	Special Condition	Special Formula	General Formula	Primary Operation
Complement not A	A^c $\sim A$	None	$P(A^c) = 1 - P(A)$		Subtraction (-)
Intersection A and B	$A \cap B$ $A \wedge B$	Independent	$P(A \cap B) = P(A)P(B)$	$P(A \cap B) = P(A)P(B A) = P(A) + P(B) - P(A \cup B)$	Multiplication (\times)
Union A or B	$A \cup B$ $A \vee B$	Mutually Exclusive	$P(A \cup B) = P(A) + P(B)$	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	Addition (+)
Conditional A given B A if B	$A B$	Independent	$P(A B) = P(A)$	$P(A B) = \frac{P(A \cap B)}{P(B)}$	Division (\div)
		Mutually Exclusive	$P(A B) = 0$		

Basic Rules of Probability

A Contingency Table

CT ← frequency Sex Party

* D I R

F 3 2 4

M 8 9 12

(~prob CT) 'D'

0.71053

'D' (^ prob CT) 'F'

0.07895

'D' (v prob CT) 'F'

0.44737

'D' (v prob CT) 'I'

0.57895

'D' (^ prob CT) 'I'

0

Some More Exotic Examples

CT

*	D	I	R
F	3	2	4
M	8	9	12

A Male or Republican

'M' (\vee prob CT) 'R'

0.86842

A Not Male or Republican

'M' ($\tilde{\vee}$ prob CT) 'R'

0.13158

A Male or Republican

A but not both

'M' (\neq prob CT) 'R'

0.55263

Conditional Probability

	CT		
*	D	I	R
F	3	2	4
M	8	9	12

⌘ Marginal Probability

$(\vdash \text{prob CT}) 'D'$

0.28947

$(\vdash \text{prob CT}) 'F'$

0.23684

⌘ Joint Probability

$'D' (\wedge \text{prob CT}) 'F'$

0.0789473684

⌘ Conditional Prob

⌘ $P(A|B) = P(A \text{ given } B)$

$'D' (| \text{prob CT}) 'F'$

0.33333

$'F' (| \text{prob CT}) 'D'$

0.27273

Set Theoretic Function Equivalents

Logical Function	Set Theoretic Function
And $P(A \wedge B)$	Intersection $P(A \cap B)$
Or $P(A \vee B)$	Union $P(A \cup B)$
A and not B $P(A > B)$	Without $P(A \sim B)$

'F' (^ prob CT) 'R'
0.10526

'F' (∩ prob CT) 'R'
0.10526

'F' (∪ prob CT) 'R'
0.55263

'F' (∪ prob CT) 'R'
0.55263

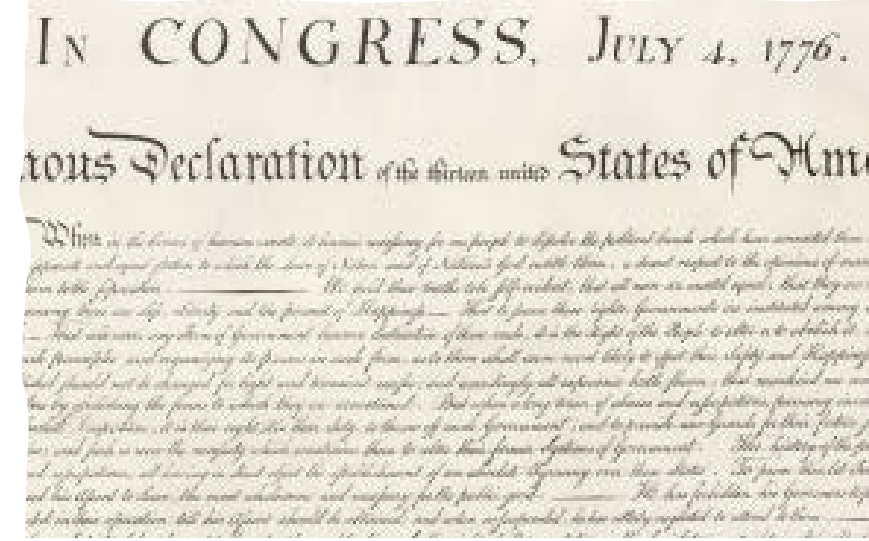
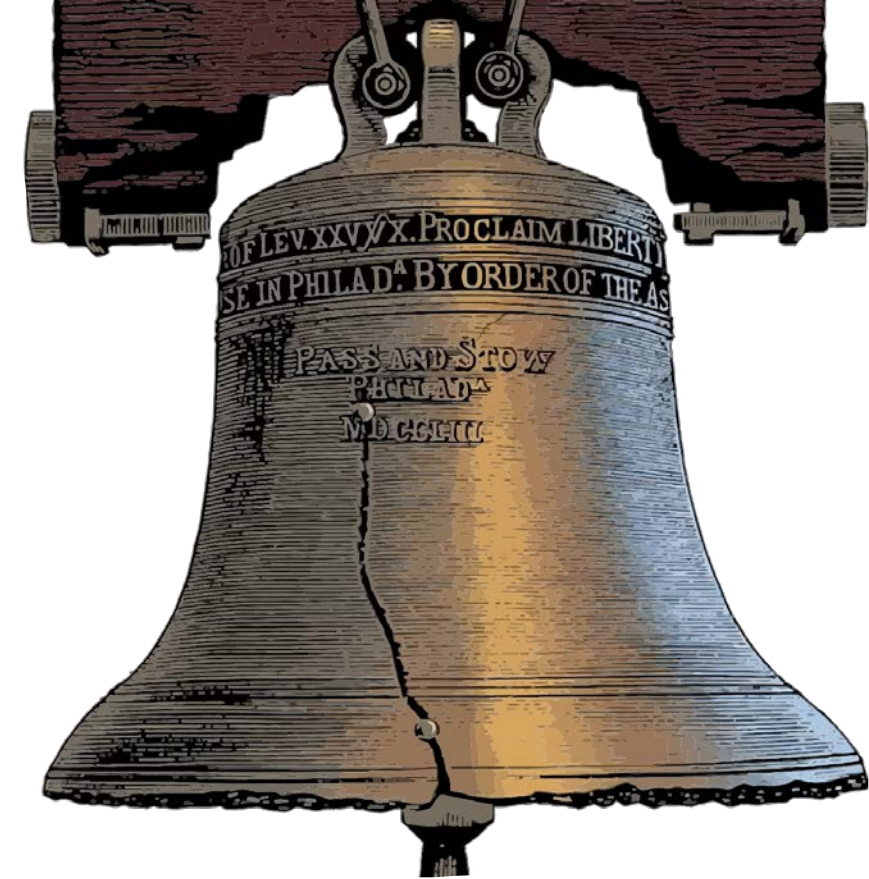
'F' (> prob CT) 'R'
0.13158

'F' (~ prob CT) 'R'
0.13158

Independence

- Two events are independent, if the occurrence of one has no effect on the occurrence of the other.
- In particular, $P(A \cap B) = P(A)P(B)$.
- To find the probability of two independent events requires only the individual marginal probabilities e.g.

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$



Using the independent (**ind**) operand

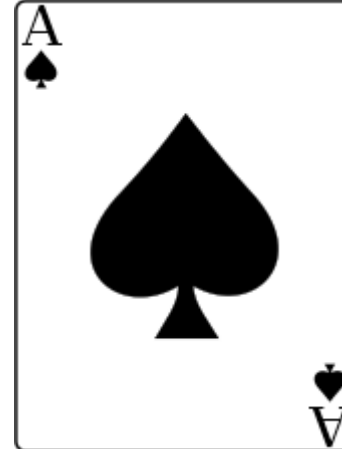
A ← 4 ÷ 52 **A** Prob(Ace)
0.076923

B ← 26 ÷ 52 **A** Prob(Black)
0.5

A Prob (Black Ace)
A (^ **prob ind**) **B**
0.038461

A Prob(Ace or Black)
A (v **prob ind**) **B**
0.53846

A Neither Ace nor black
A (ṽ **prob ind**) **B**
0.46154



Expression 'F'(^ probability Sex Party)'D' Result: 0.0789

Rows Sex 2 * D I R
 Columns Party 3 F 3 2 4
 M 8 9 12

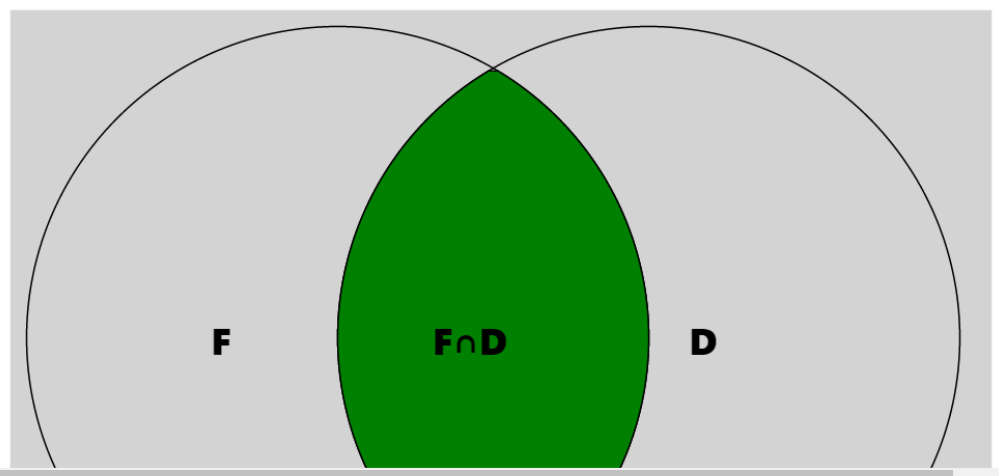
Set A F (Optional)

Conjunction ^ and

Operator probability

Right Operand Table

Set B D



^ and

- F and D 7.89%**
- Complement 92.11%**

Probability Wizard