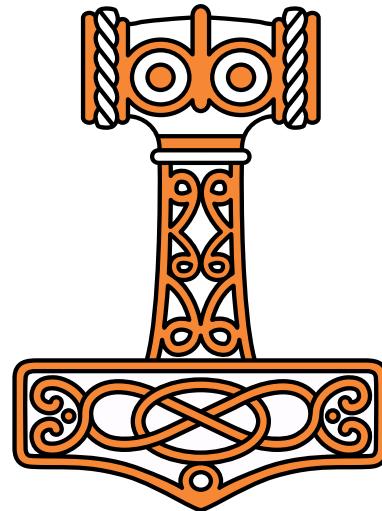


DYALOG

Olhão 2022

Filling the Core Language Gaps

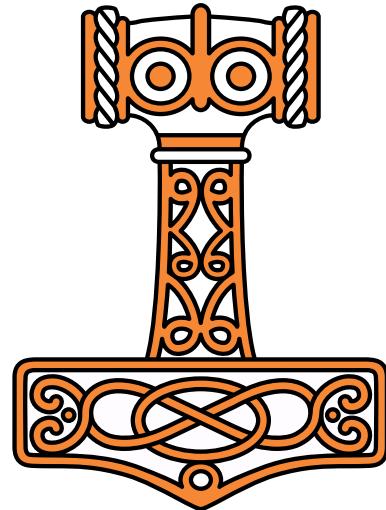
Adám Brudzewsky





Filling the Core Language Gaps

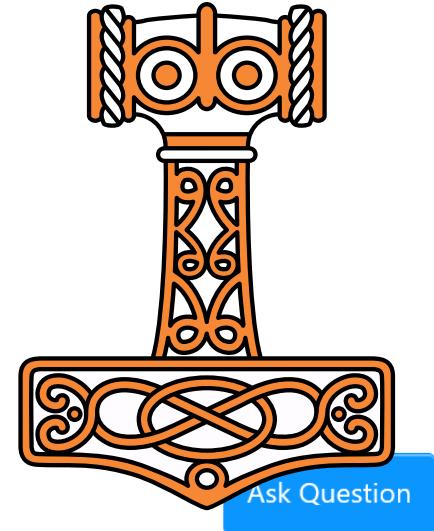
Adám Brudzewsky



Filling the Core Language Gaps

Indexing with nested vectors in APL

Asked 2 years, 3 months ago Modified 2 years, 2 months ago Viewed 132 times



Ask Question



I have a vector of vectors that contain some indices, and a character vector which I want to use them on.

3

A←(1 2 3)(3 2 1)
B←'ABC'

CC BY-SA: stackoverflow.com/q/62319267

I have a vector of vectors that contain some

3

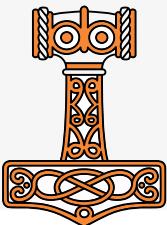
```
A<-c(1 2 3)(3 2 1)  
B<-'ABC'
```

I have tried:

```
B[A]  
RANK ERROR
```

```
B[A]
```

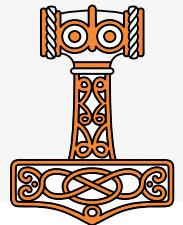
^



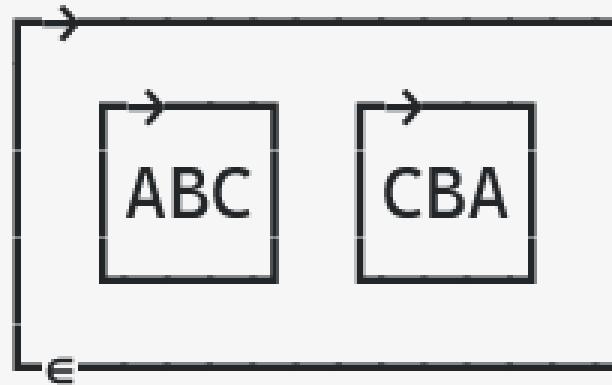
A⊗B
LENGTH ERROR
A⊗B
^

and

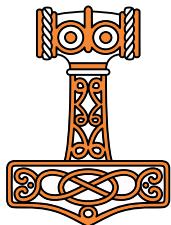
A⊗B
LENGTH ERROR
A⊗''B
^



I would like

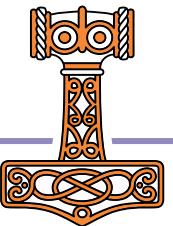


to be returned, but if i need to find another way,



Indexing with Nested Vectors

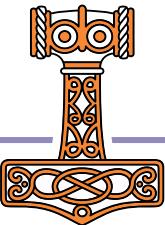
3 Answers:



Indexing with Nested Vectors

3 Answers:

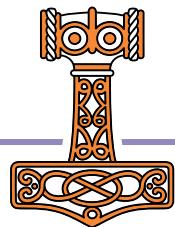
- ◆ $(\subset^\cdot A) \sqsubseteq^\cdot \subset B$



Indexing with Nested Vectors

3 Answers:

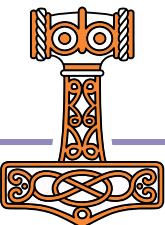
- ◆ $(\subset^\omega A) \sqsubseteq^\omega \subset B$
- ◆ $\{B[\omega]\}^{\cdots\cdots} A$



Indexing with Nested Vectors

3 Answers:

- ◆ $(\subset^\omega A) \sqsubseteq^\omega \subset B$
- ◆ $\{B[\omega]\}^{\cdots\cdots} A$
- ◆ *Don't do that!*



Indexing with Nested Vectors

3 Answers:

- $(\subset^\omega A) \sqsubseteq^\omega \subset B$

- $\{B[\omega]\}^{\cdots} A$

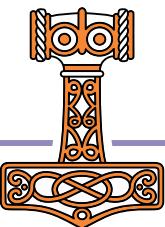
- *Don't do that!*

Possibilities:

- $A \sqsubseteq^{\cdots} \subset \subset B$

- $\sqsubseteq \circ B \sqsubseteq^\omega A$

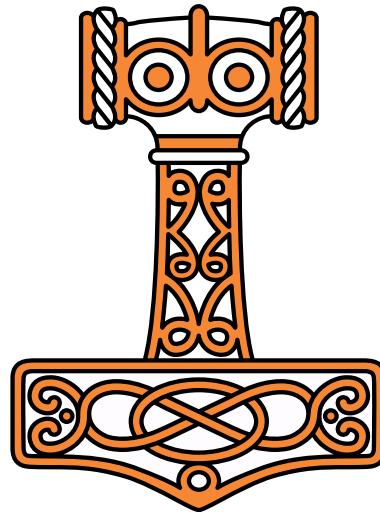
- $A (\subset^\omega \dashv \sqsubseteq) \sqsubseteq^\omega B$



DYALOG

Olhão 2022

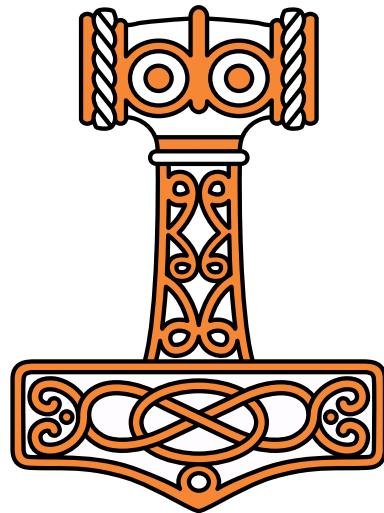
Filling the Core Language Gaps



DYALOG

Olhão 2022

Core Language

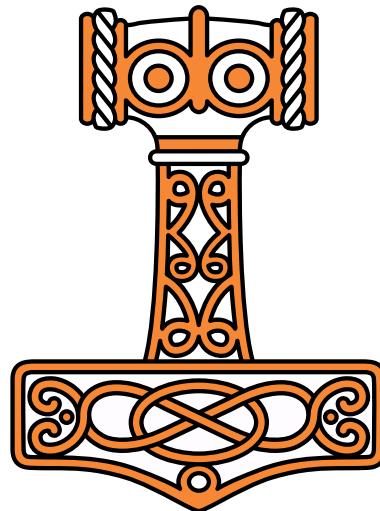




Olhão 2022

Core Language

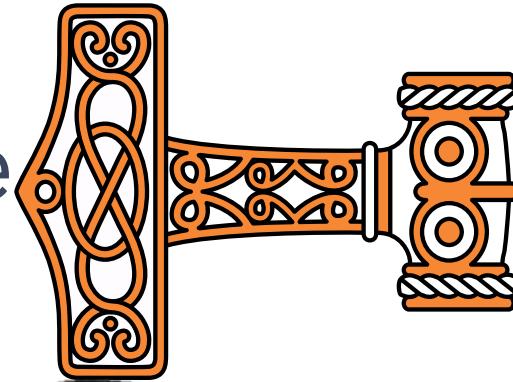
Squiggles!





Olhão 2022

Core Language



Squiggles!

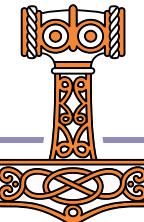
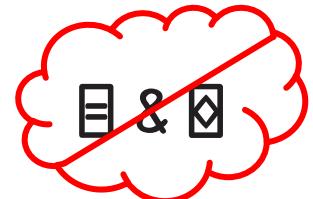


Core Language



Core Language

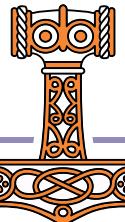
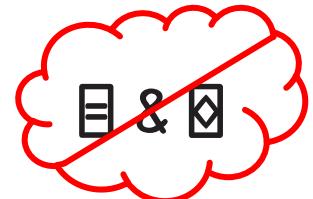
Data Transformation



Core Language

Data Transformation

Function Application

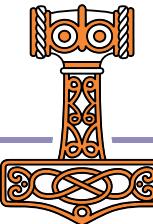
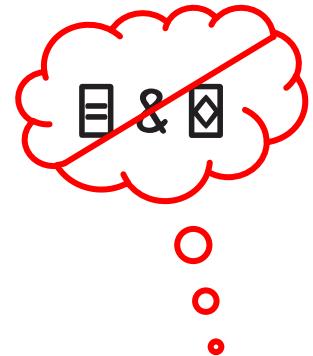


Core Language

Data Transformation

Function Application

Function Composition



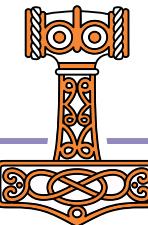
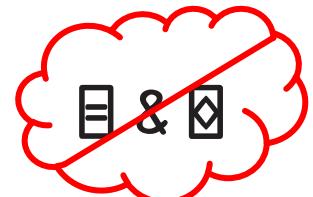
Core Language

Data Transformation

 $X \times Y$ ϕY $X \sqcap Y$

Function Application

Function Composition



Core Language

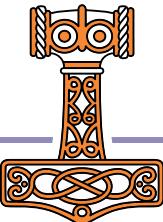
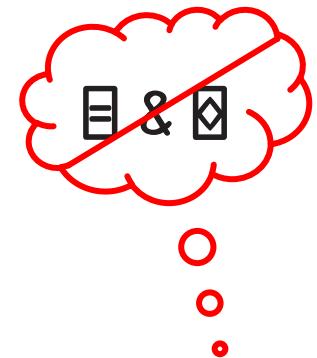
Data Transformation

 $X \times Y$ ϕY $X \sqcap Y$

Function Application

 $f \neq$ $f \ddot{*} g$ $f \ddot{o} k$

Function Composition



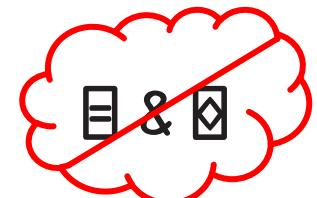
Core Language

Data Transformation

$X \times Y$

ϕY

$X \sqsubseteq Y$



Function Application

$f \neq$

$f \ddot{*} g$

$f \ddot{o} k$

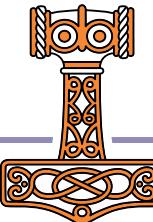


Function Composition

$f \ddot{o} g$

$f \ddot{o} g$

$f \circ g$



Core Language

Data Transformation

$X \times Y$

ϕY

$X \boxplus Y$

Function Application

$f \neq$

$f \ddot{*} g$

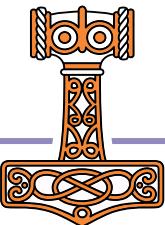
$f \ddot{o} k$

Function Composition

$f \ddot{o} g$

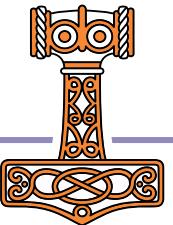
$f \ddot{o} g$

$f \circ g$



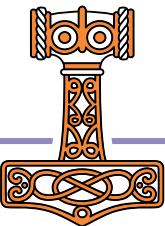
Data Transformation

Indexing



Data Transformation

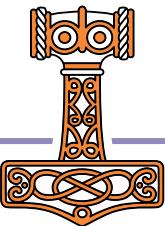
Simple Indexing



Data Transformation

Simple Indexing

Choose Indexing

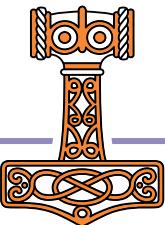


Data Transformation

Simple Indexing

Choose Indexing

Reach Indexing

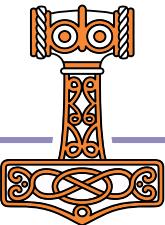


Data Transformation

- ◆ Simple Indexing

- Choose Indexing

- Reach Indexing



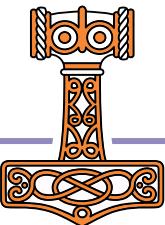
Data Transformation

- ◆ Simple Indexing

$t \leftarrow 3 \cdot 8 \rho \square A$

Choose Indexing

Reach Indexing



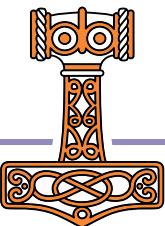
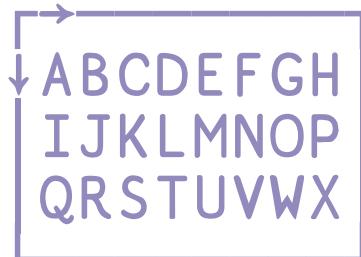
Data Transformation

- ◆ Simple Indexing

$t \leftarrow 3 \text{ } 8 \rho \square A$

Choose Indexing

Reach Indexing



Data Transformation

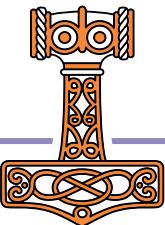
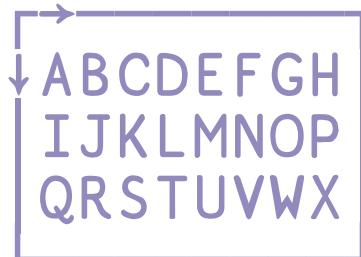
- ◆ Simple Indexing

$t \leftarrow 3 \text{ np} \square A$

- Choose Indexing

$t[2:4]$

- Reach Indexing



Data Transformation

- Simple Indexing

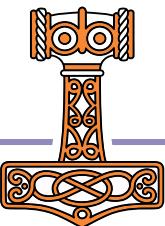
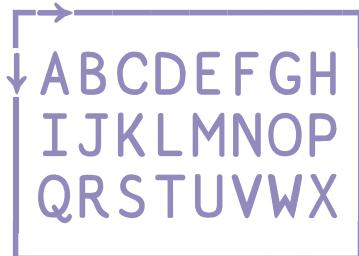
$t \leftarrow 3 \text{ np} \square A$

- Choose Indexing

$t[2:4]$

- Reach Indexing

L

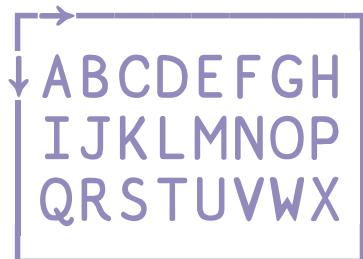


Data Transformation

- Simple Indexing

Choose Indexing

Reach Indexing



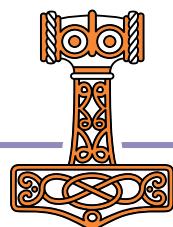
$t \leftarrow 3 \text{ np}[]A$

$t[2:4]$

L

$2 \text{ : } 4 \text{ } []t$

L

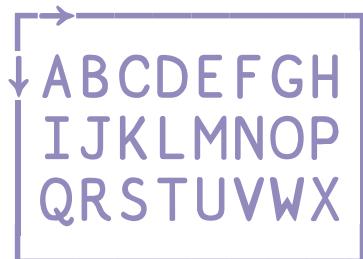


Data Transformation

- Simple Indexing

Choose Indexing

Reach Indexing



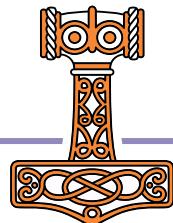
$t \leftarrow 3 \text{ } 8 \rho \square A$

$t[2 \ 1; 4 \ 1 \ 7]$

LIO
DAG

$(2 \ 1)(4 \ 1 \ 7) \square t$

LIO
DAG



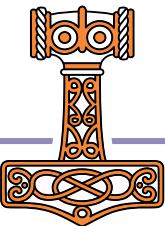
Data Transformation

- ◆ Simple Indexing

$p \leftarrow 8p \square A$

Choose Indexing

Reach Indexing



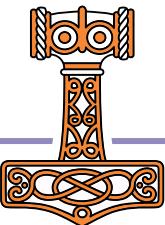
Data Transformation

- ◆ Simple Indexing

$p \leftarrow 8p \square A$

Choose Indexing

Reach Indexing



Data Transformation

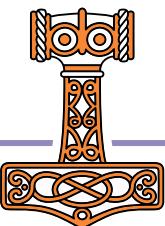
- ◆ Simple Indexing

`p←8p[]A`

- Choose Indexing

`p[2]`

- Reach Indexing



Data Transformation

- Simple Indexing

Choose Indexing

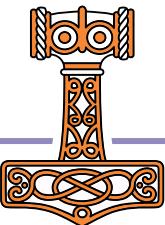
Reach Indexing



$p \leftarrow 8 \times A$

$p[2]$

B



Data Transformation

- Simple Indexing

Choose Indexing

Reach Indexing



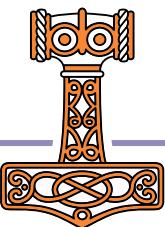
$p \leftarrow 8 \text{p} \square A$

$p[2]$

B

$2 \square p$

B



Data Transformation

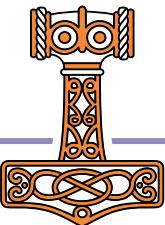
- Simple Indexing

Choose Indexing

Reach Indexing



$p \leftarrow 8$
 $p[2 \ 1 \ 7]$
BAG



Data Transformation

- Simple Indexing

Choose Indexing

Reach Indexing



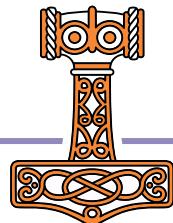
$p \leftarrow 8$ $p[] A$

$p[2 \ 1 \ 7]$

BAG

2 \ 1 \ 7?p

BAG

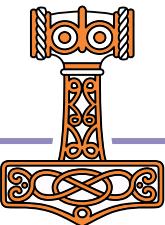


Data Transformation

Simple Indexing

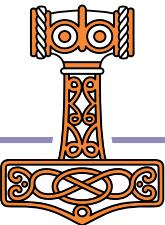
- ◆ Choose Indexing

- Reach Indexing



Data Transformation

- Simple Indexing $t \leftarrow 3 \text{ } 8 \rho \square A$
- ◆ Choose Indexing
- Reach Indexing

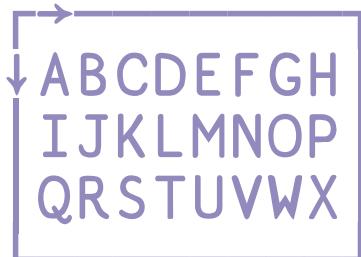


Data Transformation

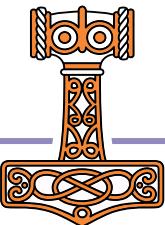
Simple Indexing

- ◆ Choose Indexing

Reach Indexing



$t \leftarrow 3 \text{ } 8 \rho \square A$

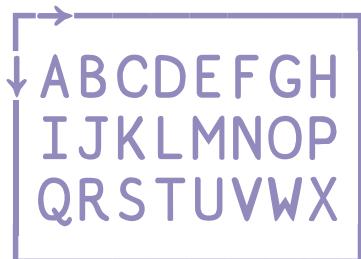


Data Transformation

Simple Indexing

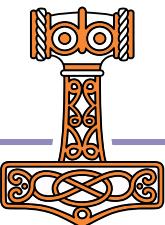
- ◆ Choose Indexing

Reach Indexing



$t \leftarrow 3 \text{ } 8 \rho \square A$

$t[<1 \text{ } 8]$

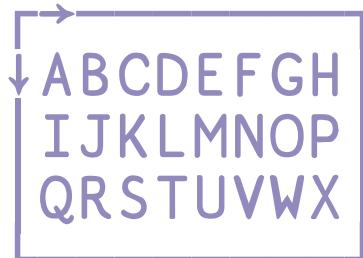


Data Transformation

Simple Indexing

- ◆ Choose Indexing

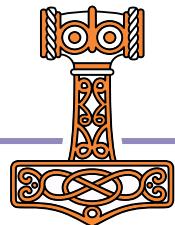
Reach Indexing



`t ← 3 8 p[]A`

`t[<1 : 8]`

H

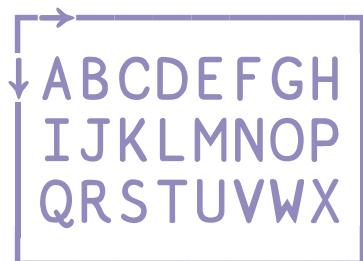


Data Transformation

Simple Indexing

◆ Choose Indexing

Reach Indexing



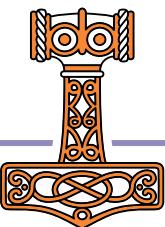
`t ← 3 8 p[]A`

`t[<1 8]`

H

`1 8 []t`

H

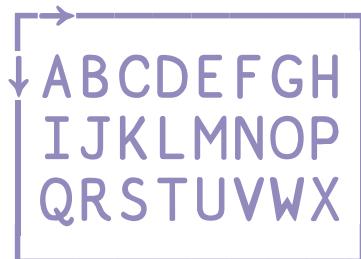


Data Transformation

Simple Indexing

◆ Choose Indexing

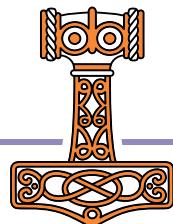
Reach Indexing



`t[3:8]`

`t[(1:8)(2:7)]`

HO

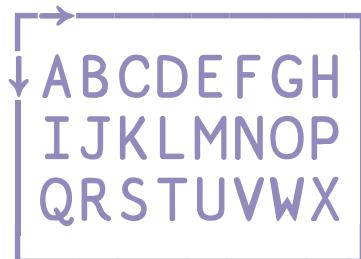


Data Transformation

Simple Indexing

- Choose Indexing

- Reach Indexing



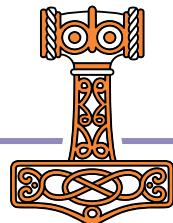
$t \leftarrow 3 \text{ np} \square A$

$t[(1\ 8)(2\ 7)]$

HO

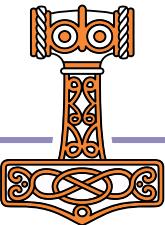
HO

$(1\ 8)(2\ 7)?t$



Data Transformation

- Simple Indexing
- Choose Indexing
- ◆ Reach Indexing



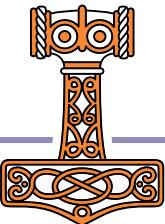
Data Transformation

Simple Indexing

```
s<- 'Dad' 'Mom', -3 5
```

Choose Indexing

- ◆ Reach Indexing

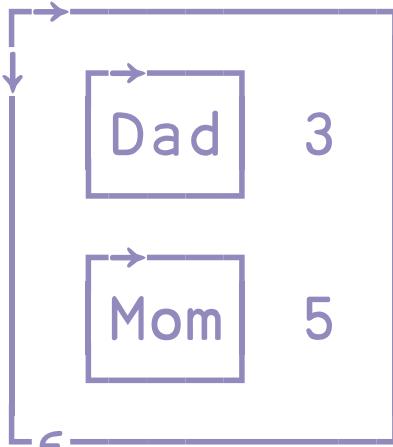


Data Transformation

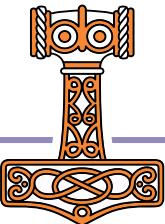
Simple Indexing

Choose Indexing

- Reach Indexing



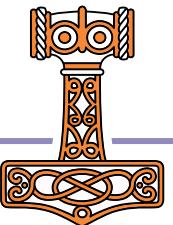
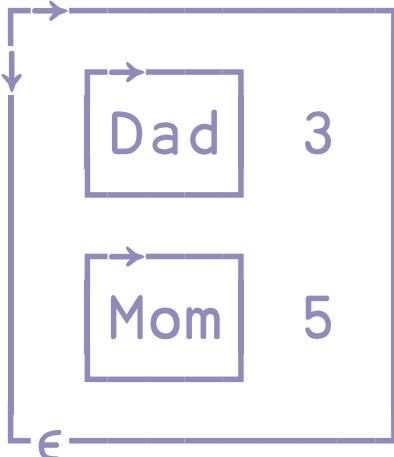
```
s<- 'Dad' 'Mom', 3 5
```



Data Transformation

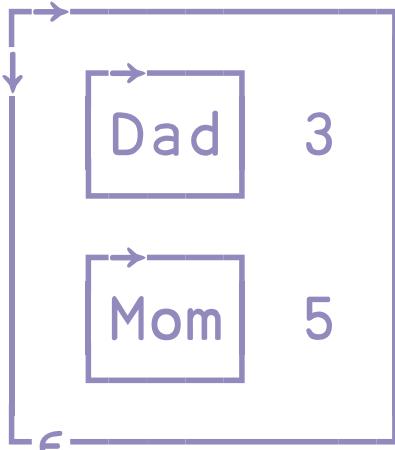
- Simple Indexing
- Choose Indexing
- Reach Indexing

```
s<- 'Dad' 'Mom', -3 5  
s[ c(2 1)3 ]
```



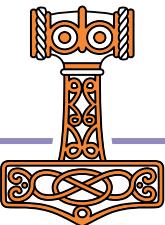
Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



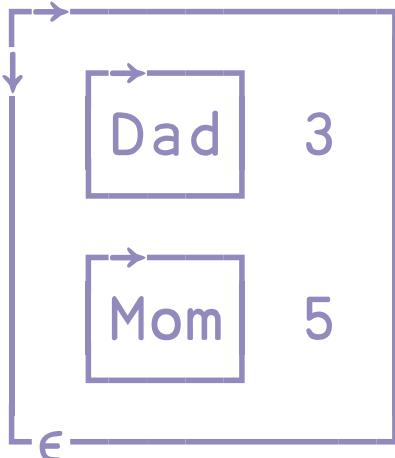
m

`s ← 'Dad' 'Mom', ;3 5`
`s[<(2 1) 3]`



Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



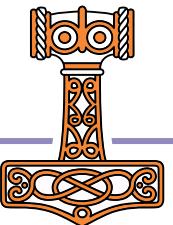
`s ← 'Dad' 'Mom', 3 5`

`s[<(2 1)3]`

m

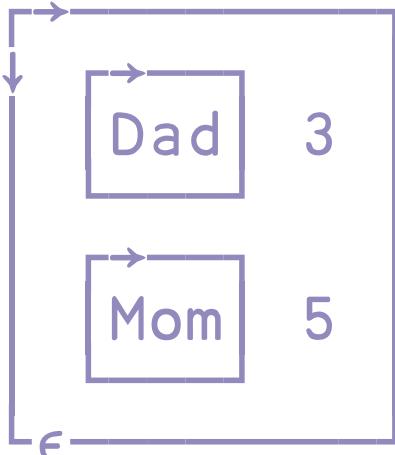
m

`(2 1)3 ⇒ s`



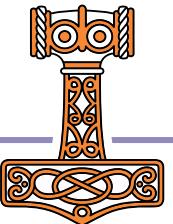
Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



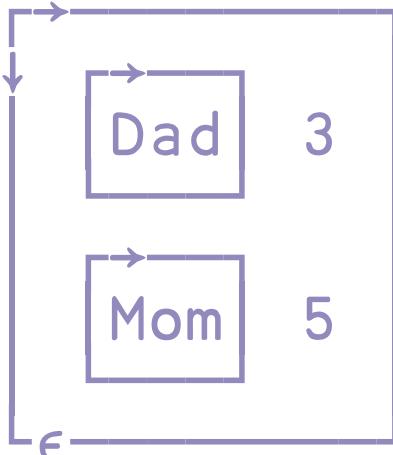
`s<- 'Dad' 'Mom', -3 5`
`s[c(2 1)3]`

m



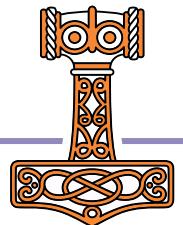
Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing



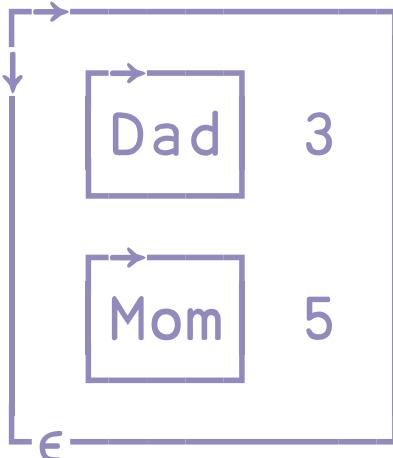
ma

```
s<- 'Dad' 'Mom', -3 5  
s[ ((2 1)3)((1 1)2)]
```



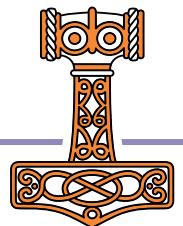
Data Transformation

- Simple Indexing
- Choose Indexing
- Reach Indexing

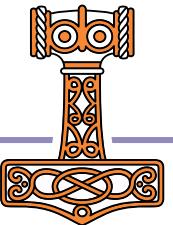


ma
ma

```
s<- 'Dad' 'Mom', -3 5  
s[((2 1)3)((1 1)2)]  
((2 1)3)((1 1)2)?s
```



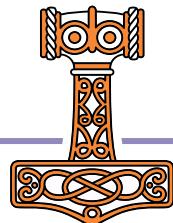
Data Transformation



Data Transformation

Select

X ⊃ Y



Indexing with Nested Vectors

3 Answers:

- $(\subset^\sim A) \sqsubseteq^\sim \subset B$

- $\{B[\omega]\}^{\sim\sim\sim} A$

- *Don't do that!*

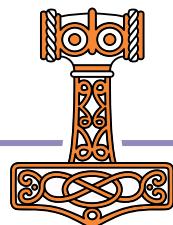
Possibilities:

- $A \sqsubseteq^\sim \subset B$

- $\sqsubseteq^\sim B \subset^\sim A$

- $A (\subset^\sim \dashv \sqsubseteq) \subset B$

With Select $X \sqsupseteq Y$:



Indexing with Nested Vectors

3 Answers:

- $(\subset^\sim A) \emptyset^\sim \subset B$

- $\{B[\omega]\}^\sim \sim A$

- *Don't do that!*

Possibilities:

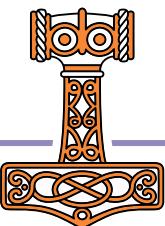
- $A \emptyset^\sim \sim \subset \subset B$

- $\emptyset \circ B \sim \subset^\sim A$

- $A (\subset^\sim \dashv \emptyset \vdash) \sim \subset B$

With Select $X \exists Y$:

- $A \exists^\sim \subset B$



Indexing with Nested Vectors

3 Answers:

- $(\subset^\sim A) \mathbb{I}^\sim \subset B$

- $\{B[\omega]\}^\sim \sim A$

- *Don't do that!*

Possibilities:

- $A \mathbb{I}^\sim \subset \subset B$

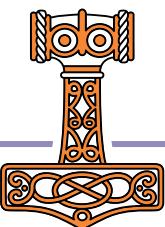
- $\mathbb{I} \circ B^\sim \subset^\sim A$

- $A (\subset^\sim \dashv \mathbb{I} \vdash)^\sim \subset B$

With Select $X \sqsupseteq Y$:

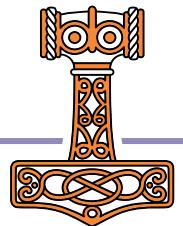
- $A \sqsupseteq^\sim \subset B$

- $\sqsupseteq \circ B^\sim \subset A$



Data Transformation

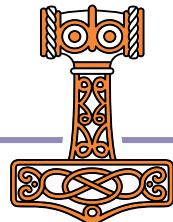
X Ø Y Index



Data Transformation

X ⊸ Y Index

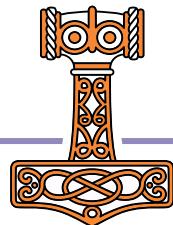
- ◆ Sort $\leftarrow (\text{c} \ddot{\circ} \Delta \square \vdash)$



Data Transformation

X ⌢ Y Index

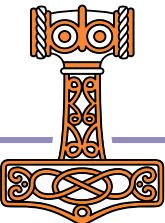
- ◆ Sort $\leftarrow (\text{c} \ddot{\circ} \Delta \square \vdash)$
- ◆ Sorts $\leftarrow \square \ddot{\circ} \circ \text{c} \circ \Delta \ddot{\circ}$ ↞ "sort Y by X"



Data Transformation

X ⌂ Y Index

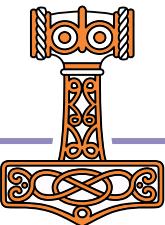
- ◆ Sort $\leftarrow (\text{c} \ddot{\circ} \text{A} \square \vdash)$
- ◆ Sorts $\leftarrow \text{I} \ddot{\circ} \circ \text{c} \circ \text{A} \ddot{\circ}$ ↗ "sort Y by X"
- ◆ Shuffle $\leftarrow (\text{c} \ddot{\circ} ? \ddot{\circ} \circ \text{f} \square \vdash)$



Data Transformation

X ⚡ Y Index

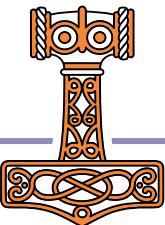
- ◆ Sort $\leftarrow (\text{c} \ddot{\circ} \Delta \square \vdash)$
- ◆ Sorts $\leftarrow \square \ddot{\circ} \circ \text{c} \circ \Delta \ddot{\circ}$ & "sort Y by X"
- ◆ Shuffle $\leftarrow (\text{c} \ddot{\circ} ? \ddot{\circ} \circ \neq \square \vdash)$
- ◆ Grade $\leftarrow ((\text{c} \text{bounds} \circ \underline{l}) \square \text{grades} \ddot{\circ})$



Data Transformation

X \ni Y Select

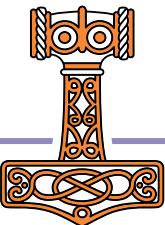
- ◆ Sort $\leftarrow (\text{c} \ddot{\circ} \Delta \square \vdash)$
- ◆ Sorts $\leftarrow \square \ddot{\circ} \circ \text{c} \circ \Delta \ddot{\circ}$ ↗ "sort Y by X"
- ◆ Shuffle $\leftarrow (\text{c} \ddot{\circ} ? \ddot{\circ} \circ \neq \square \vdash)$
- ◆ Grade $\leftarrow ((\text{c} \text{bounds} \circ \underline{\text{l}}) \square \text{grades} \ddot{\circ})$



Data Transformation

X \sqsupseteq Y Select/Permute

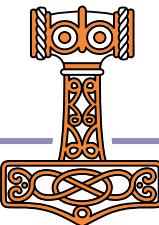
- ◆ Sort $\leftarrow (\text{c} \ddot{\circ} \text{A} \square \vdash)$
- ◆ Sorts $\leftarrow \square \ddot{\circ} \circ \text{c} \circ \text{A} \ddot{\circ}$ & "sort Y by X"
- ◆ Shuffle $\leftarrow (\text{c} \ddot{\circ} ? \ddot{\circ} \circ \text{f} \square \vdash)$
- Grade $\leftarrow ((\text{c} \text{bounds} \circ \underline{\text{l}}) \square \text{grades} \ddot{\circ})$



Data Transformation

X \ni Y Select/Permute

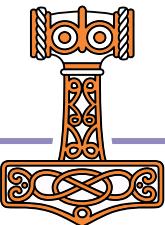
- ◆ Sort $\leftarrow (\Delta \ni \vdash)$
- ◆ Sorts $\leftarrow \exists^{\sim} \circ \Delta^{\sim}$ ↗ "sort Y by X"
- ◆ Shuffle $\leftarrow (?)^{\sim} \circ \not\in \ni \vdash$
- ◆ Grade $\leftarrow (bounds \circ \underline{I} \ni grades^{\sim})$



Data Transformation

$X \ni Y$ Select/Permute

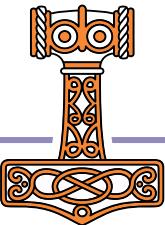
- ◆ Sort $\leftarrow (\Delta \ni \vdash)$
- ◆ Sorts $\leftarrow \exists \dots \Delta \exists$ ↗ "sort Y by X"
- ◆ Shuffle $\leftarrow (? \exists \dots \neq \ni \vdash)$
- ◆ Grade $\leftarrow (\text{bounds} \circ \underline{\text{grades}} \exists)$



Indexing with Nested Vectors

I have

```
A<-c(1 2 3)(3 2 1) ◊ B<-'ABC'
```

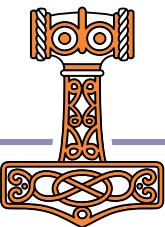
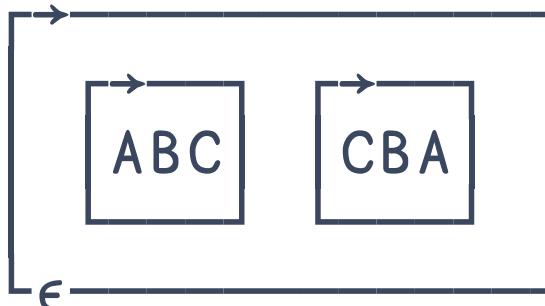


Indexing with Nested Vectors

I have

```
A<-c(1 2 3)(3 2 1) ◊ B<-'ABC'
```

I would like



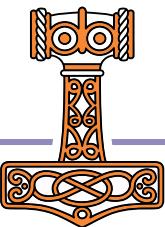
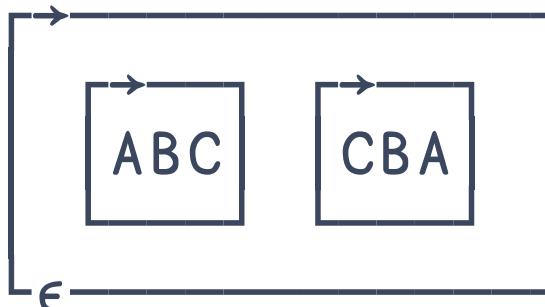
Indexing with Nested Vectors

I have

$$A \leftarrow (1 \ 2 \ 3) (3 \ 2 \ 1) \diamond B \leftarrow ABC$$

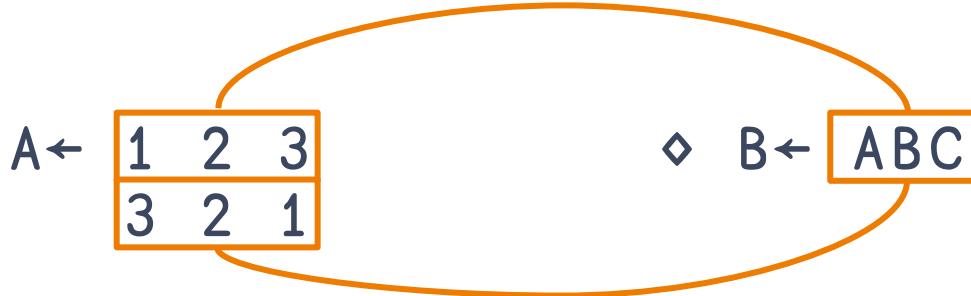
The diagram illustrates the computation of the hadamard product of two row vectors. The first vector $(1 \ 2 \ 3)$ and the second vector $(3 \ 2 \ 1)$ are enclosed in orange boxes. An orange oval encloses the entire expression $A \leftarrow (1 \ 2 \ 3) (3 \ 2 \ 1) \diamond B \leftarrow ABC$. A curved arrow points from the result of the multiplication to the string ABC , indicating that the scalar value 12 is used as an index to extract the character 'B'.

I would like

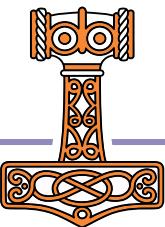


Indexing with Nested Vectors

I have

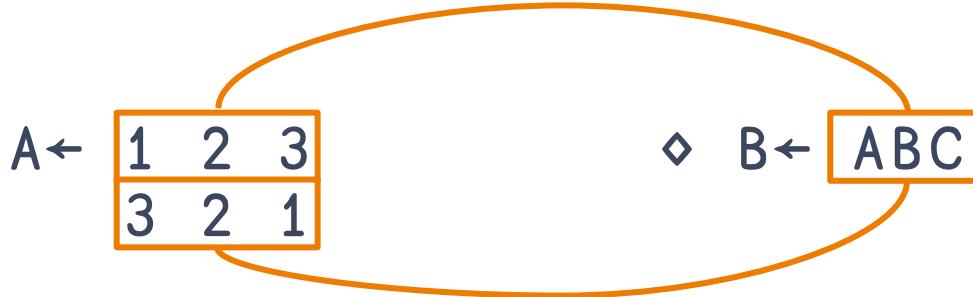


I would like



Indexing with Nested Vectors

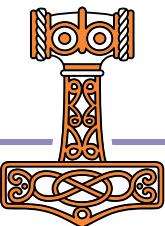
I have



I would like

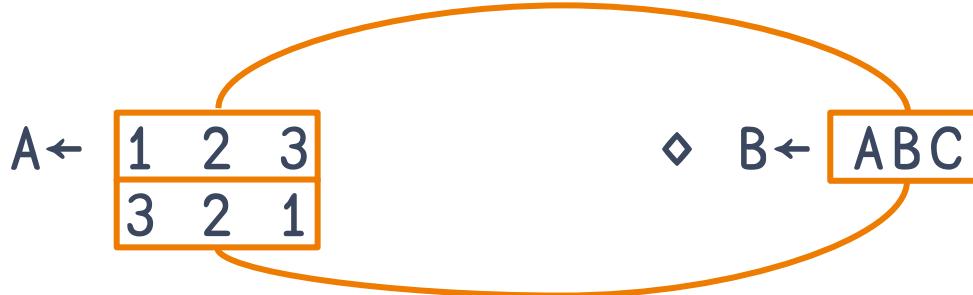


$B[A]$

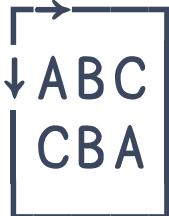


Indexing with Nested Vectors

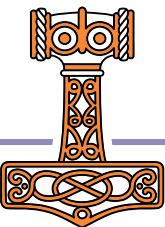
I have



I would like

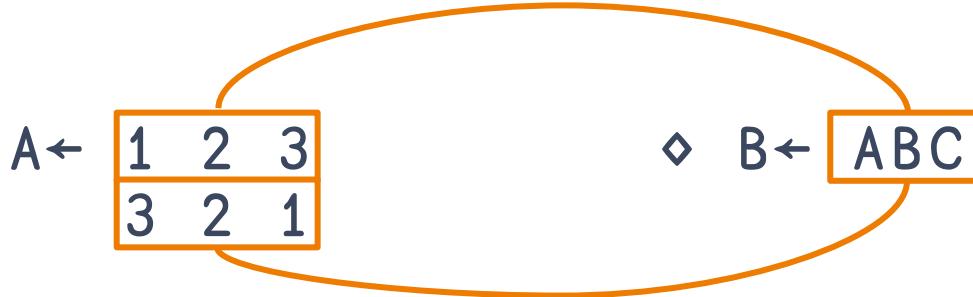


$$A \supseteq B$$



Indexing with Nested Vectors

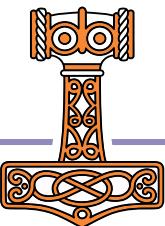
I have



I would like

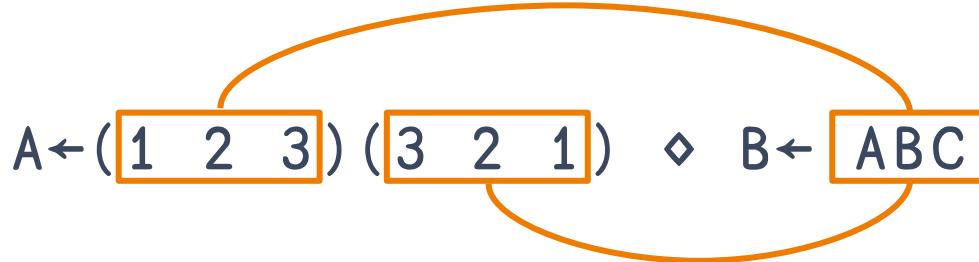


$A(_ \circ 1)B$

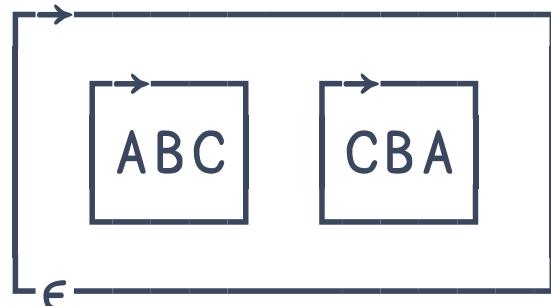
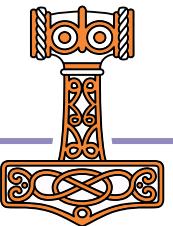


Function Application

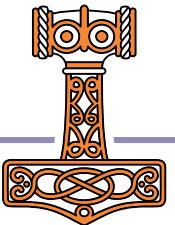
I have



I would like


$$A (\exists ? 1) B$$


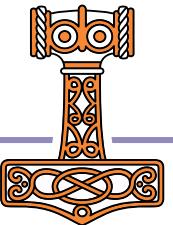
Function Application



Function Application

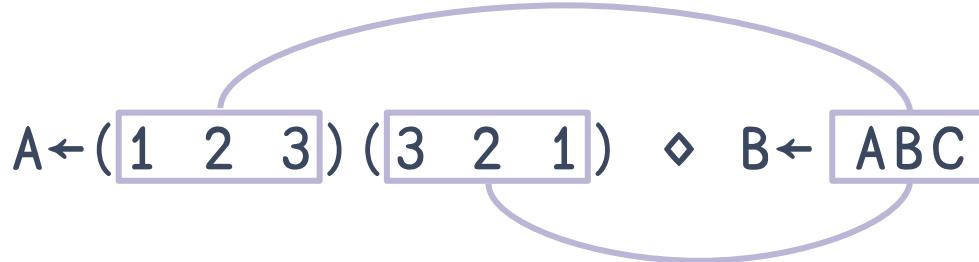
Depth

f ö k

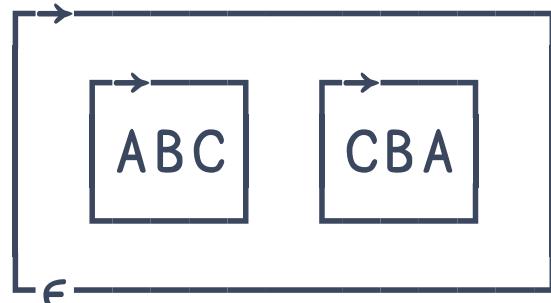


Function Application

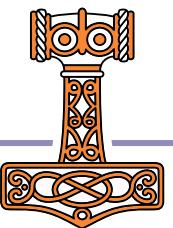
I have



I would like

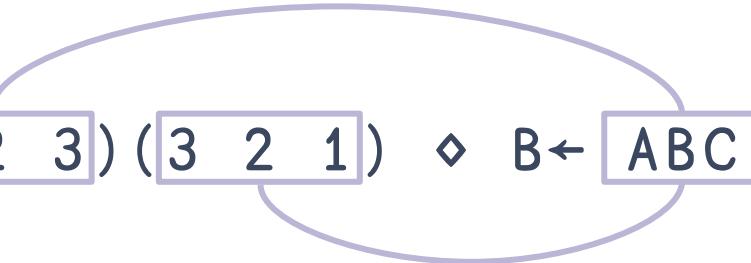


$A(\underline{\quad} \ 1)B$

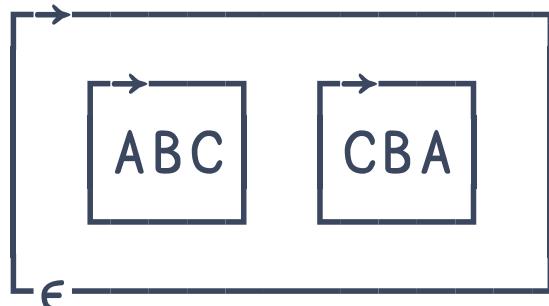


Function Application

I have

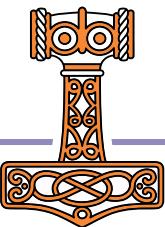
$$A \leftarrow (1 \ 2 \ 3) (3 \ 2 \ 1) \diamond B \leftarrow ABC$$


I would like



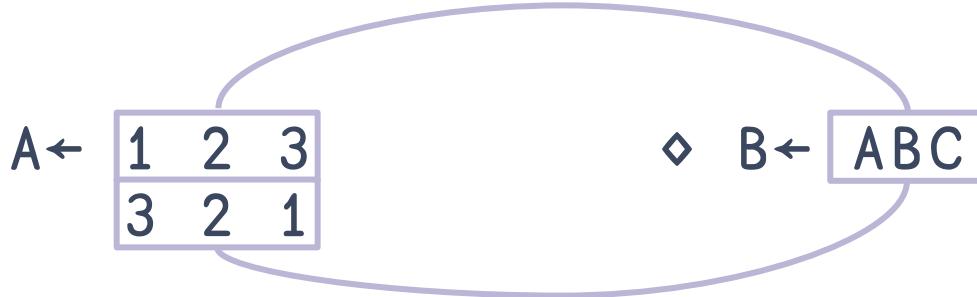
A (ε 1) B

Watch this!

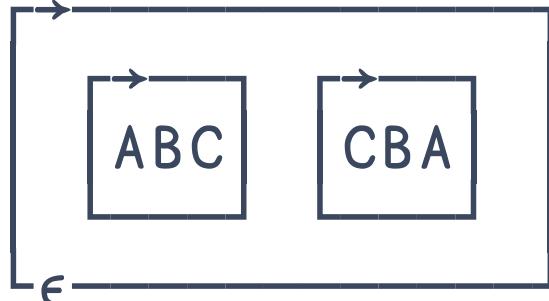


Function Application

I have

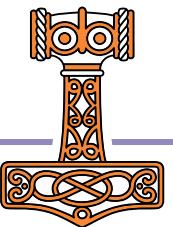


I would like



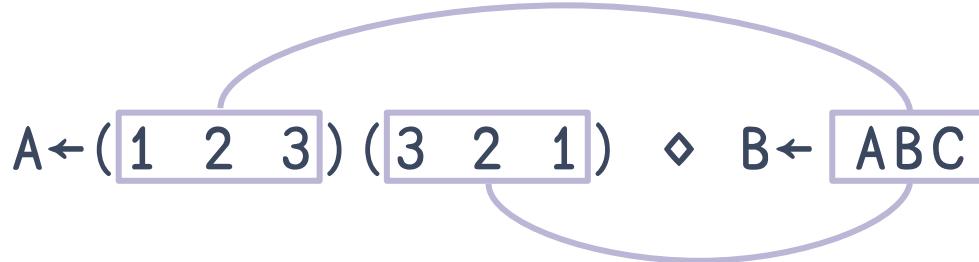
$A(_ \circ 1)B$

Watch this!

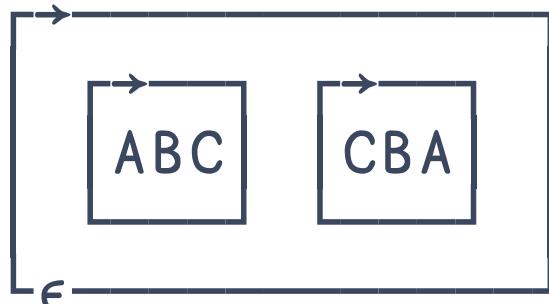


Function Application

I have

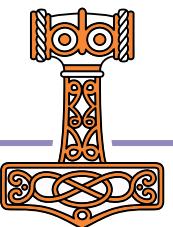


I would like



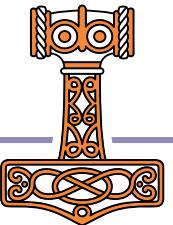
$A(\underline{\exists} \ddot{o} 1)B$

Watch this!



Function Application

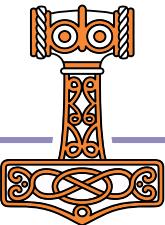
```
F p←!  
F p 4 (5 6)
```



Function Application

```
F p←!  
Fp 4 (5 6)
```

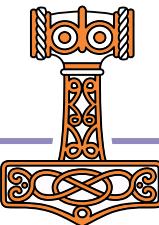
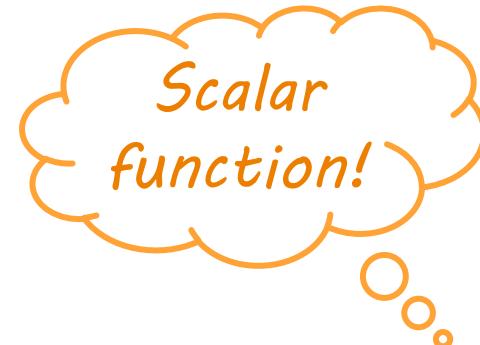
24	120	720
----	-----	-----



Function Application

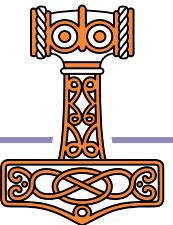
```
F p←!  
Fp 4 (5 6)
```

24	120	720
----	-----	-----



Function Application

F d←{×/⍳ω}

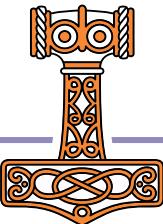


Function Application

```
Fd←{×/ᵻω}  
Fd 4 (5 6)
```

DOMAIN ERROR

```
Fd[0] Fd←{×/ᵻω}  
^
```



Function Application

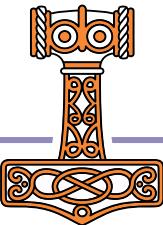
```
Fd←{×/ᵻω}  
Fd 4 (5 6)
```

DOMAIN ERROR

```
Fd[0] Fd←{×/ᵻω}
```

^

```
Fs←{×/ᵻω}ö0
```



Function Application

Fd $\leftarrow\{\times/\iota\omega\}$

Fd 4 (5 6)

DOMAIN ERROR

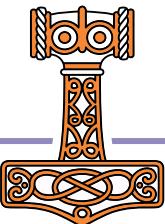
Fd[0] Fd $\leftarrow\{\times/\iota\omega\}$

^

Fs $\leftarrow\{\times/\iota\omega\} \ddot{o} 0$

Fs 4 (5 6)

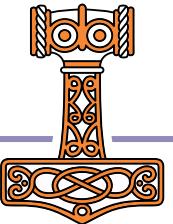
24	120	720
----	-----	-----



Function Application

$\lambda \leftarrow t_1 \leftarrow 'hi'$

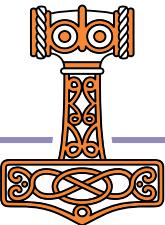
hi



Function Application

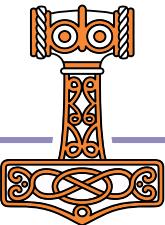
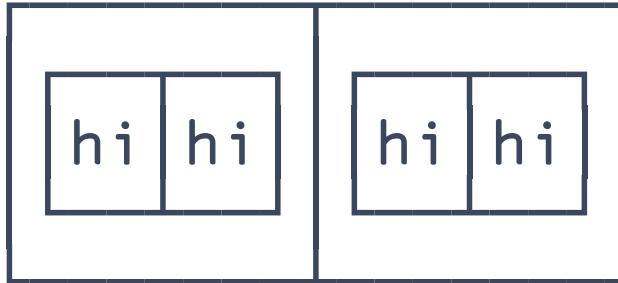
`□←t2←2ρ< t1←'hi '`

hi	hi
----	----



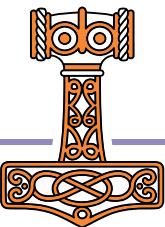
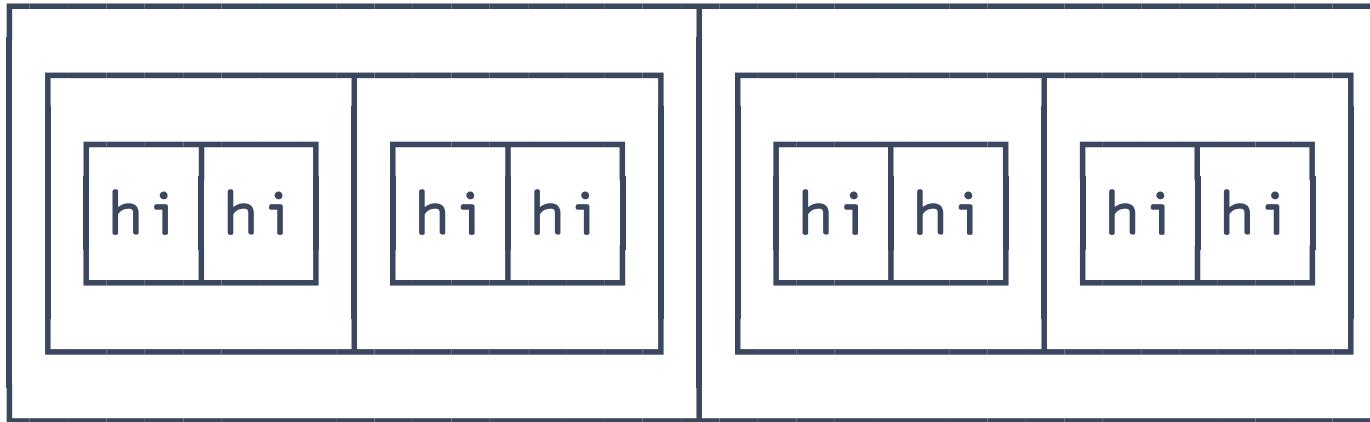
Function Application

```
l←t3←2ρct2←2ρct1←'hi '
```



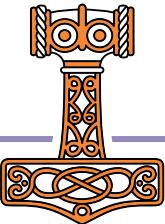
Function Application

```
0←t4←2⍴⍨t3←2⍴⍨t2←2⍴⍨t1←'hi '
```



Function Application

```
t4←2⍴t3←2⍴t2←2⍴t1←'hi'
```

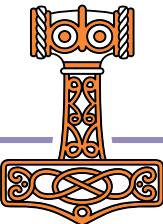


Function Application

```
t4←2⍪t3←2⍪t2←2⍪t1←'hi '
```

```
'^.' '□R' \u&'↑t1
```

Hi

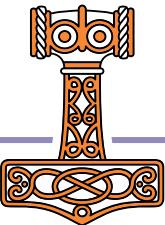


Function Application

```
t4←2⍴t3←2⍴t2←2⍴t1←'hi '
```

```
'^.' '□R' \u&'↑t2
```

Hi	Hi
----	----



Function Application

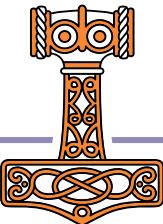
```
t4←2⍪t3←2⍪t2←2⍪t1←'hi '
```

```
'^.'□R'\u&'⊣t3
```

DOMAIN ERROR: Invalid input source

```
'^.'□R'\u&'⊣t3
```

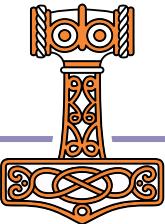
^



Function Application

```
t4←2⍪t3←2⍪t2←2⍪t1←'hi '
```

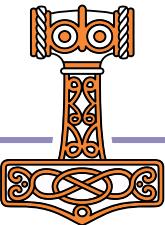
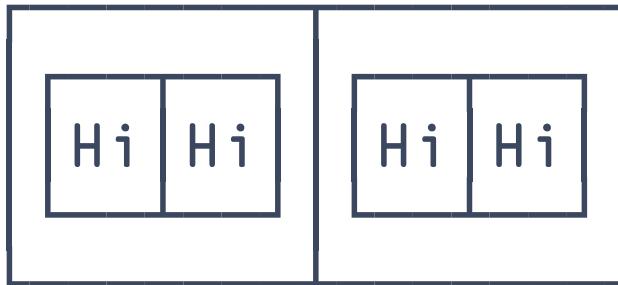
```
'^.' '□R' \u&'ö2←t3
```



Function Application

```
t4←2⍪t3←2⍪t2←2⍪t1←'hi '
```

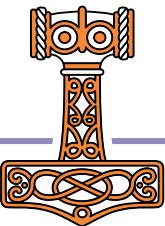
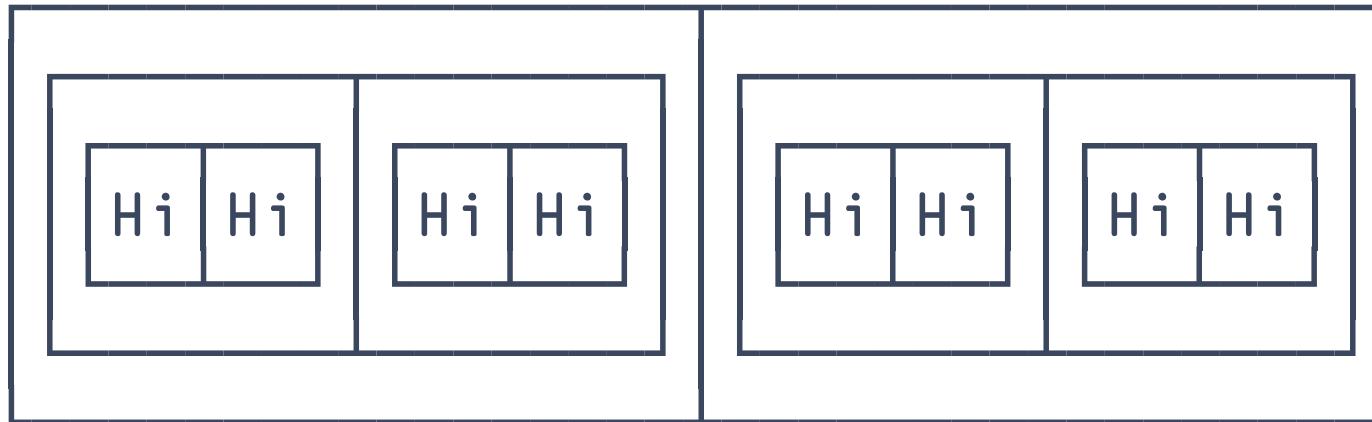
```
'^.' '□R' '\u&' ö2←t3
```



Function Application

```
t4←2⍴t3←2⍴t2←2⍴t1←'hi '
```

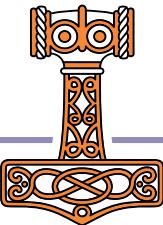
```
'^.' '□R' \u&'ö2←t4
```



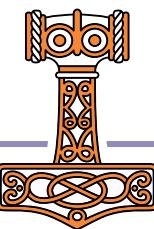
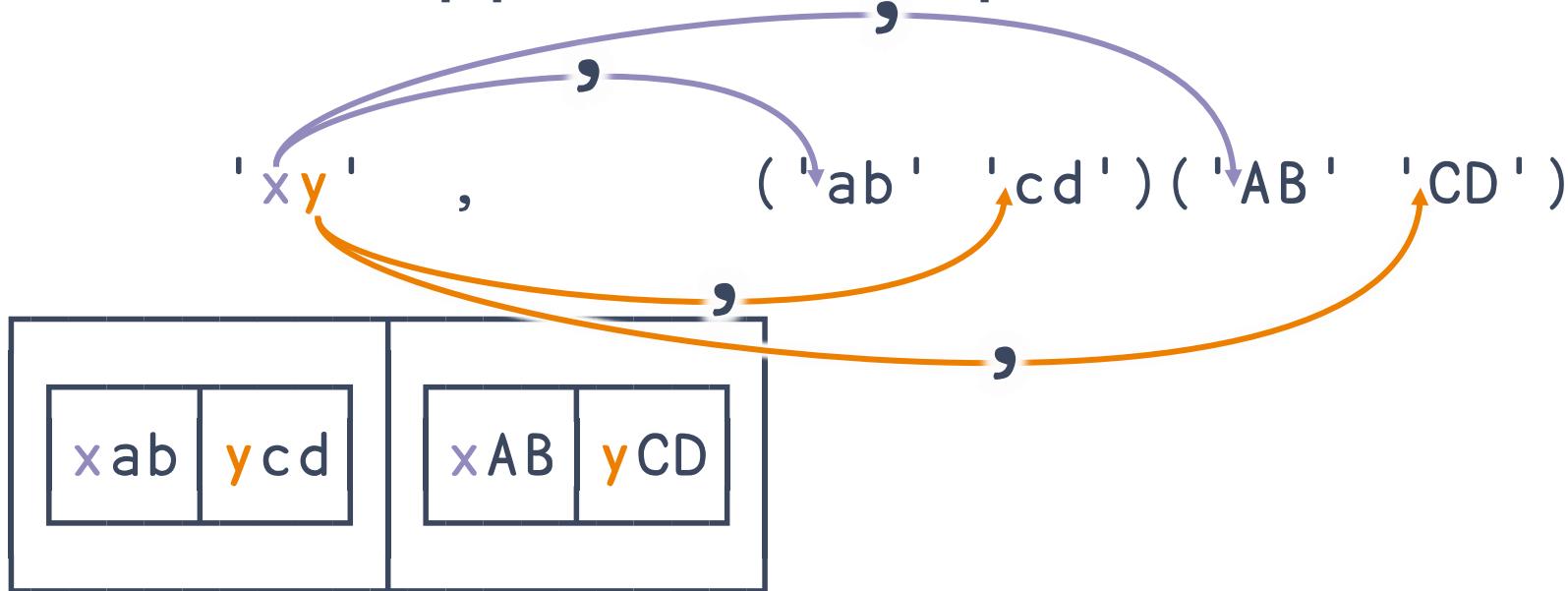
Function Application

```
t4←2ρ←t3←2ρ←t2←2ρ←t1←'hi '
```

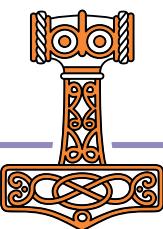
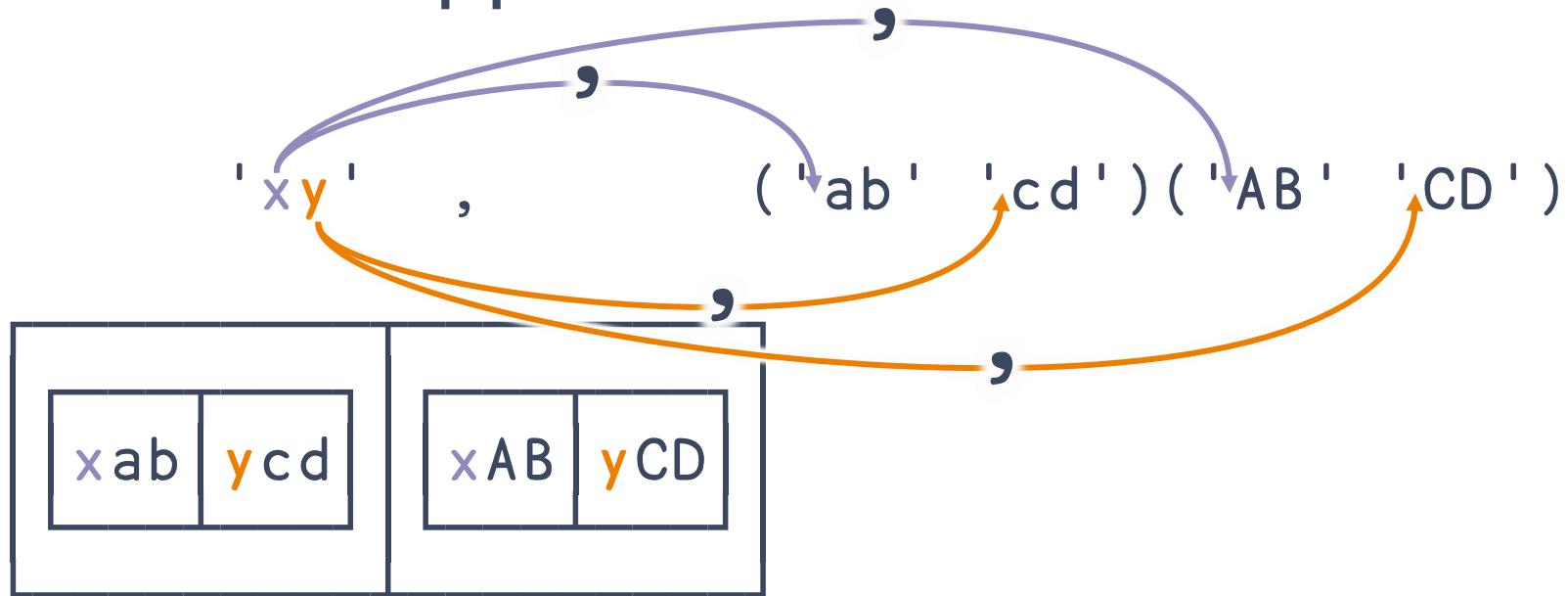
```
'^.' □R '\u00f62←t4  
{2≤|≡ω:∇''ω ◊ '^.' □R '\u00f6'←ω}
```



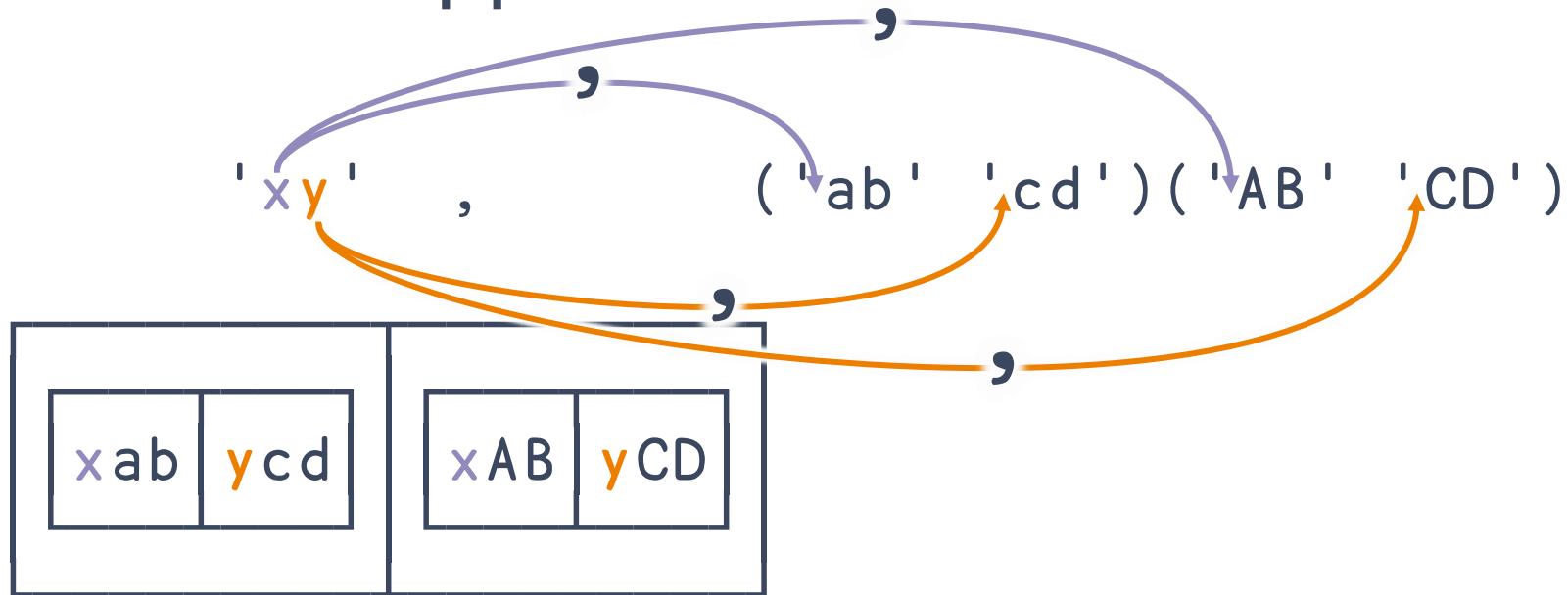
Function Application – quiz



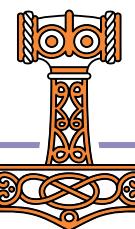
Function Application



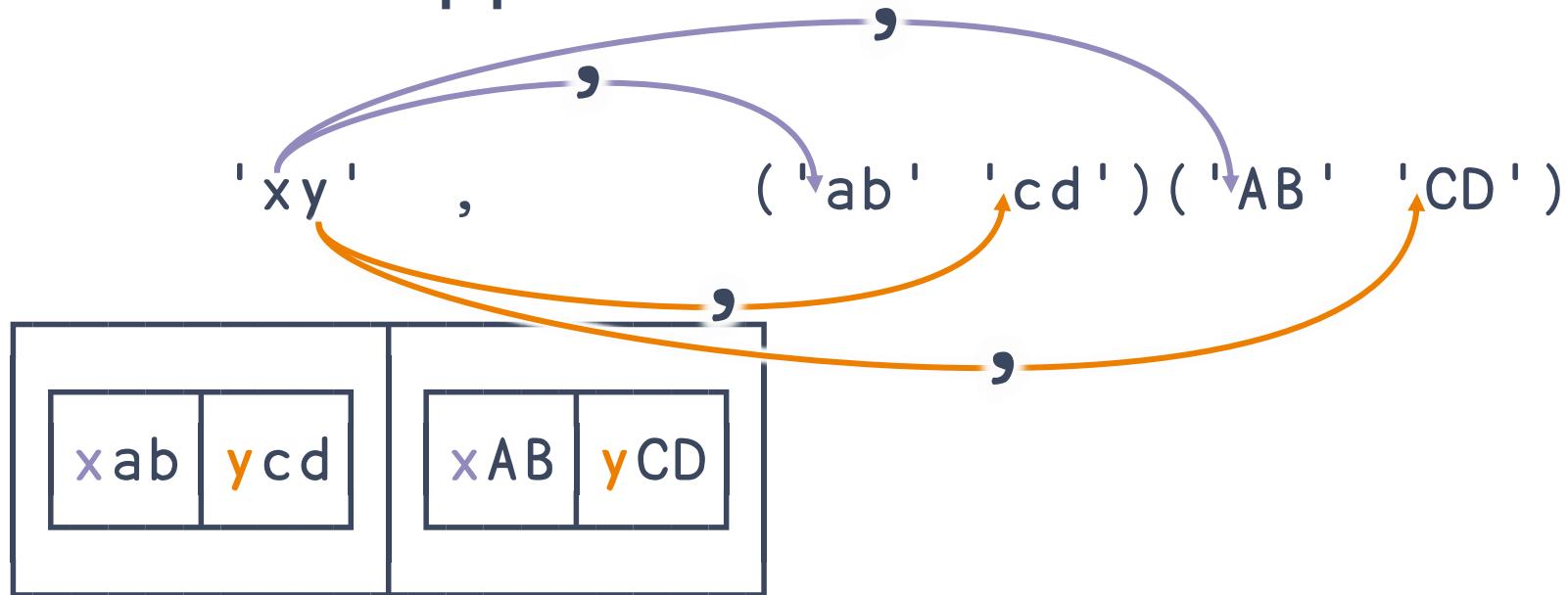
Function Application



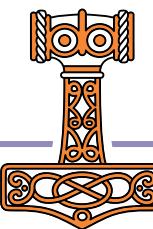
$(< 'xy') , \dots , 'xy' (, \dots ö1 2)$ $('ab' 'cd')('AB' 'CD')$ $('ab' 'cd')('AB' 'CD')$



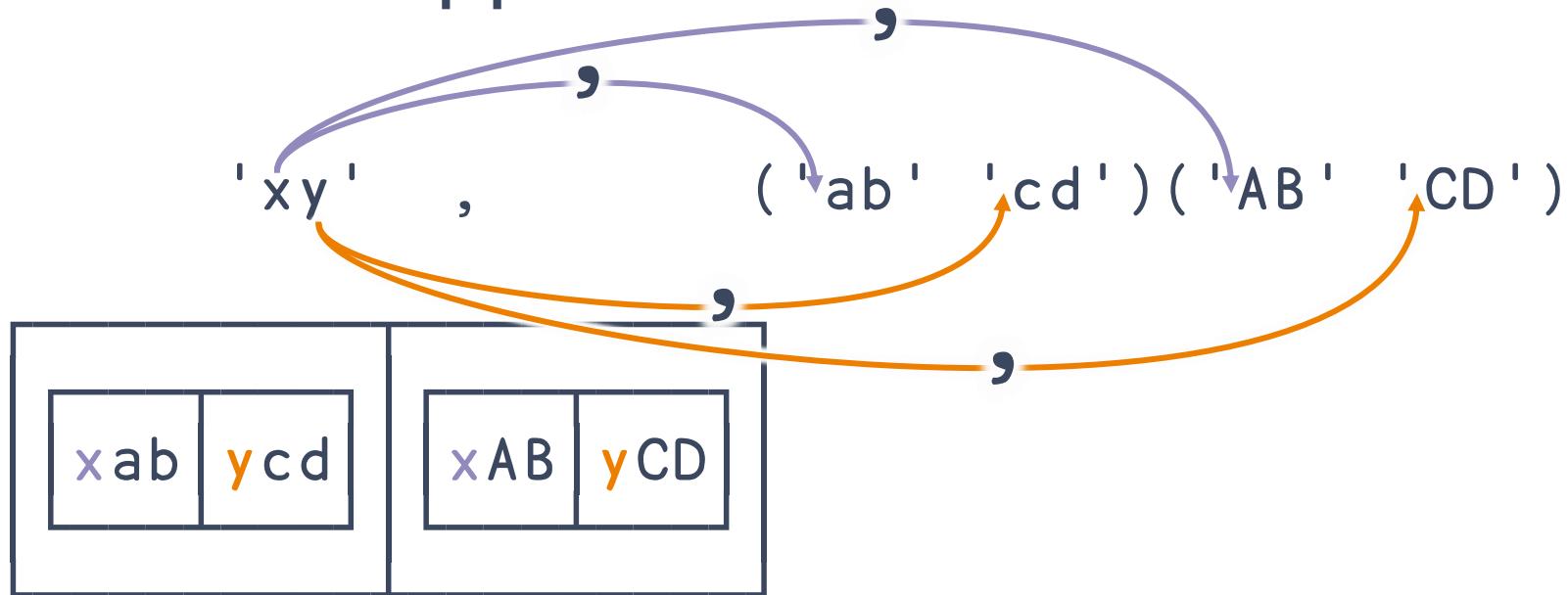
Function Application



$(< 'xy') , \dots$
 $'xy' (, \dots)$ $('ab' 'cd') ('AB' 'CD')$
 $('ab' 'cd') ('AB' 'CD')$



Function Application



$(('ab' \ 'cd')('AB' \ 'CD')) \ ('ab' \ 'cd')('AB' \ 'CD')$



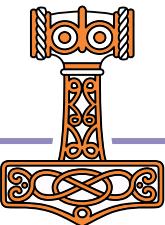
Core Language

Data Transformation

Function Application

f ö k

Function Composition



Core Language

Data Transformation

$X \times Y$

ϕY

$X \sqcap Y$

$X \sqsupseteq Y$

Function Application

$f \neq$

$f \ddot{*} g$

$f \ddot{o} k$

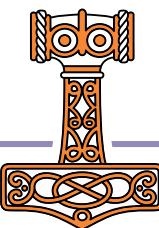
$f \ddot{e} k$

Function Composition

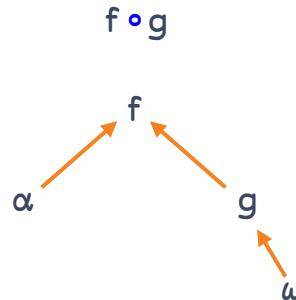
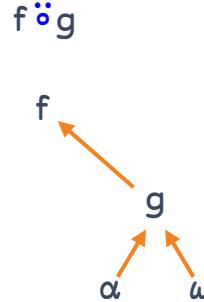
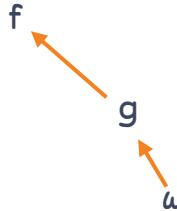
$f \ddot{o} g$

$f \ddot{o} g$

$f \circ g$

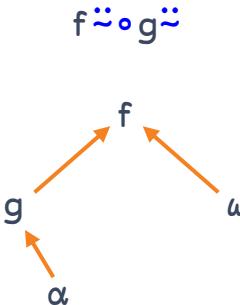


f ög f °g f ög

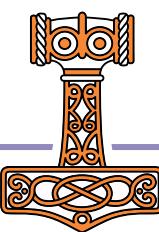
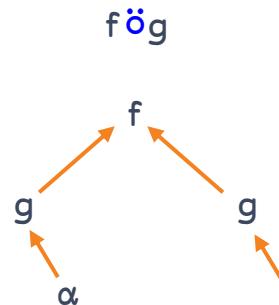


Function Composition

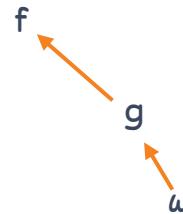
f °g



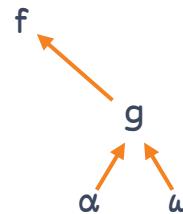
f ög



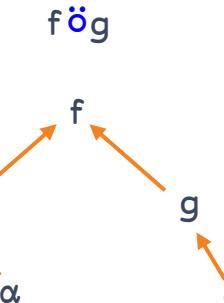
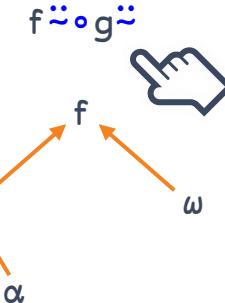
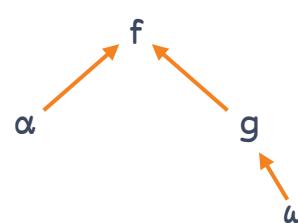
f ög f °g f ög



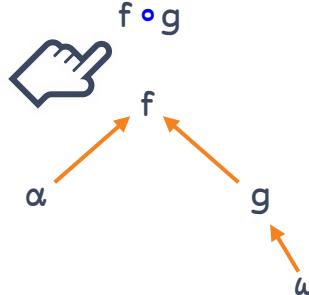
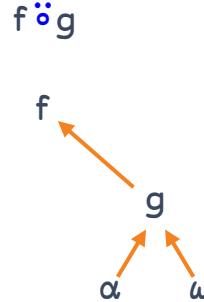
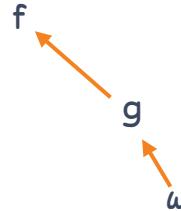
f ög



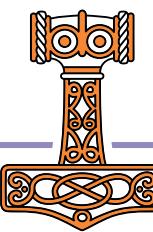
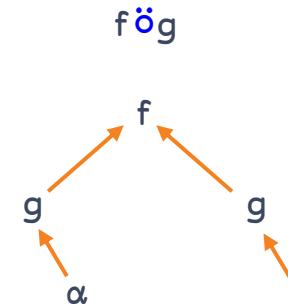
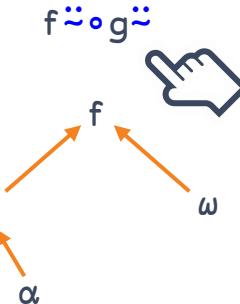
f °g



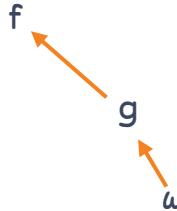
f ög f °g f ög



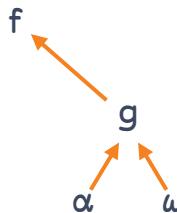
Function Composition



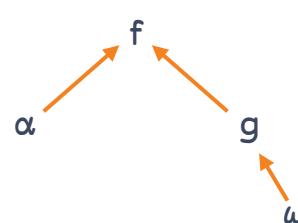
f ög f o g f ög



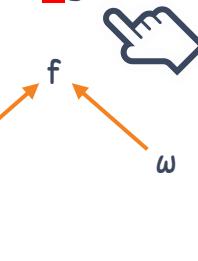
f ög



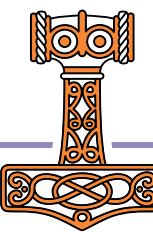
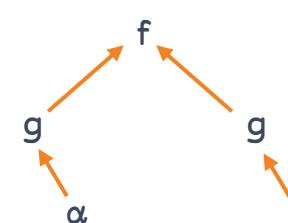
f o g



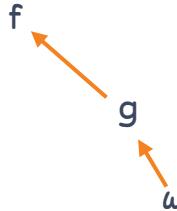
f ? g



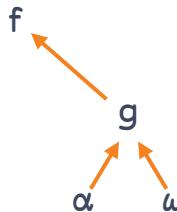
f ög



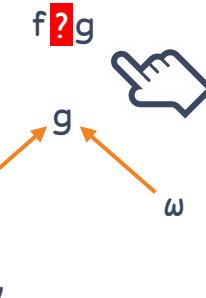
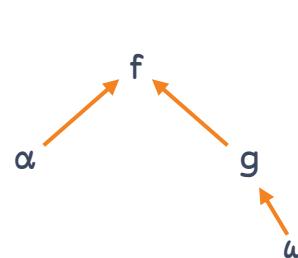
f ög f o g f ög



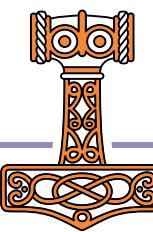
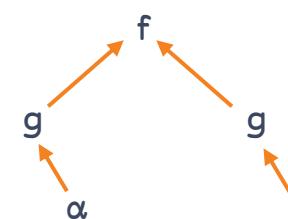
f ög



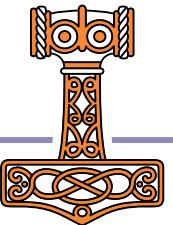
f o g



f ög



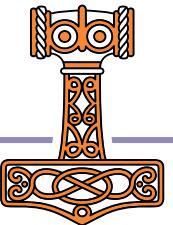
Function Composition



Function Composition

Behind

f o g

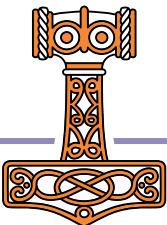


Function Composition

Behind

$f \circ g$

$X(f \circ g)Y$

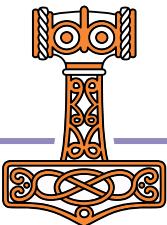


Function Composition

Behind

$f \circ g$

$x (f \quad) g \quad y$

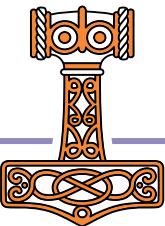


Function Composition

Behind

$f \circ g$

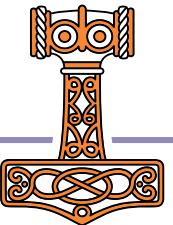
$(f \ X)g \ Y$



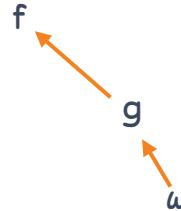
Function Composition

Behind

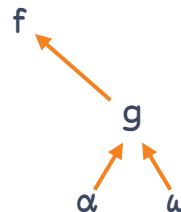
f o g



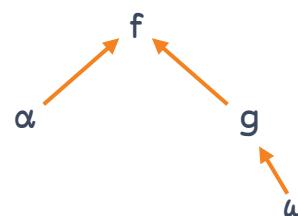
f ög f o g fög



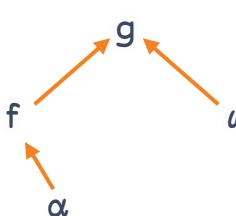
f ög



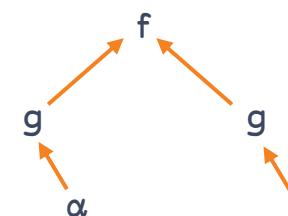
f o g



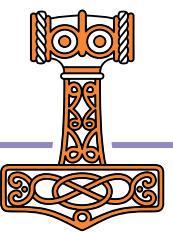
f o g



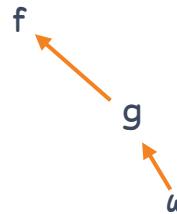
f ög



Function Composition

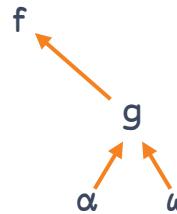


f ög f o g fög

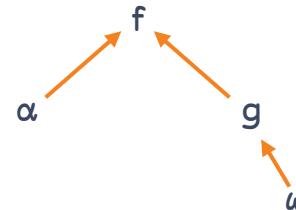


Function Composition

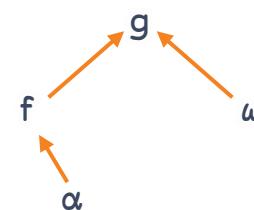
f ög



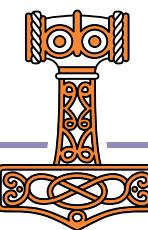
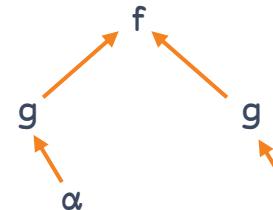
f o g



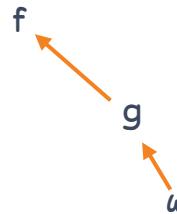
f o g



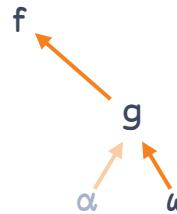
fög



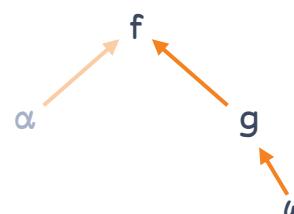
f ög f o g fög



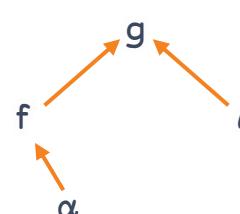
f ög



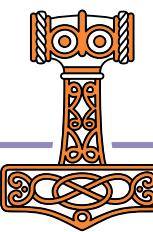
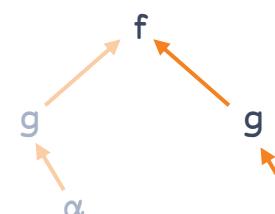
f o g



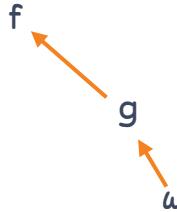
f o g



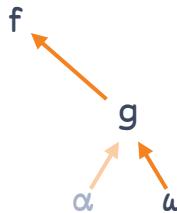
f ög



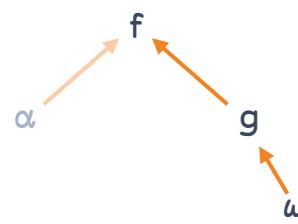
f ög f o g fög



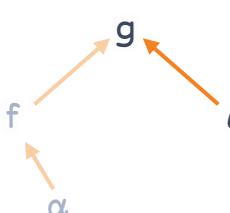
f ög



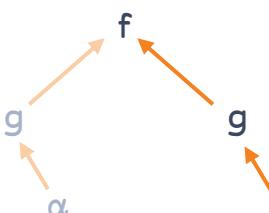
f o g



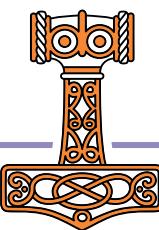
f o g



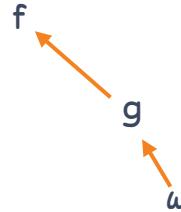
f ög



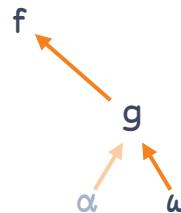
Function Composition



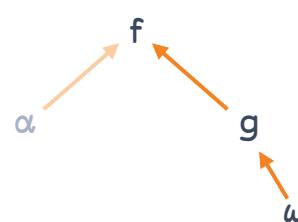
f ög f °g fög



f ög



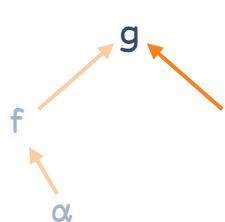
f °g



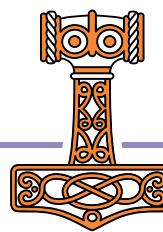
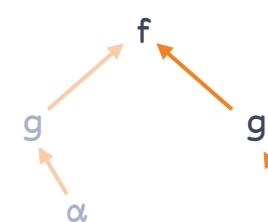
Function Composition

f og $\omega \Leftrightarrow$ g ω

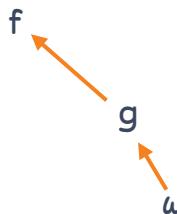
f °g



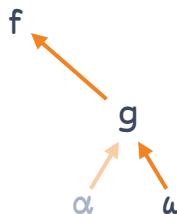
f ög



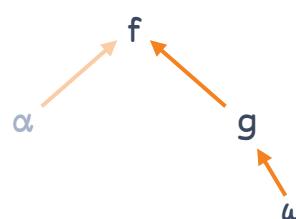
f ög f ög f ög



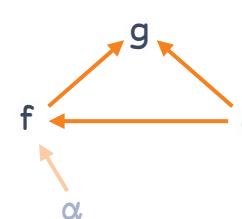
f ög



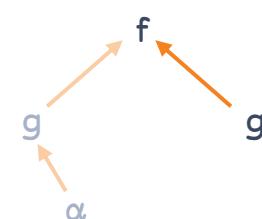
f ög



f ög



f ög



f o g ω \Leftrightarrow f o g \circ ω





Essays/Hook Conjunction?

< Essays

Hook is a 2-train, an isolated sequence of two verbs, introduced in APL by K.E. Iverson and E.E. McDonnell, *Phrasal Forms*, APL89, APL Quote-Quad, Volume 19, Number 4, 1989-08. It is defined as follows:

```
(g h) y   +  y g h y
x (g h) y   +  x g h y
```

For example, the monad `(=<.)` is a test for integers and `(+%)`/ computes a continued fraction -- `(+%)/20$1` is an approximation of the golden ratio. Hook is based on the S combinator of [combinatory logic](#).

With over 17 years of hindsight, I believe it would have been better to use a conjunction (denoted by `h.`, say) to denote a hook rather than using a 2-train. Everything that can be done with the 2-train `(f g)` can be done with the conjunction `h.`, but `h.` does not require a special parsing rule.

The original motivation for assigning a meaning to a train of length 2 was so that a train of any length (greater than 1) would be interpreted: A train with odd length is a sequence of forks; a train with even length is either a hook (if of length 2) or a hook followed by a sequence of forks (if of length >2). Again with hindsight, the alternatives are:

0. Leave trains of even length uninterpreted -- just signal error.
1. Assign the "at" meaning to it:

```
(g h) y   +  g   h y
x (g h) y   +  g x h y
```

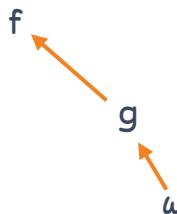
That is, the [capped fork](#) meaning. You'd probably still have the capped fork. Compare:

```
[: f0 [: f1 f2 f3 f4
(f0 (f1 (f2 f3 f4)))
```

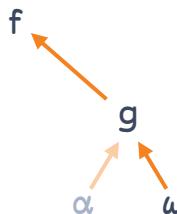
See also

- [Trains](#)

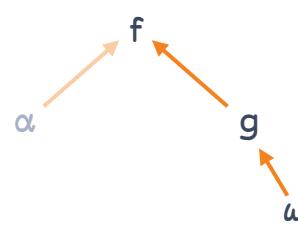
f ög f o g fög



f ög

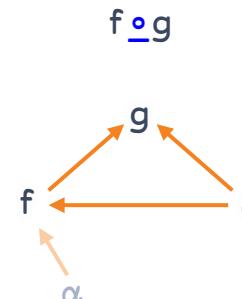


f o g

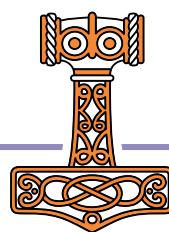
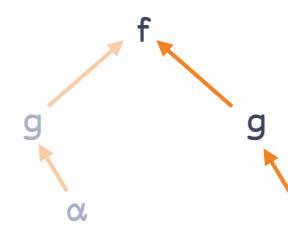


Function Composition

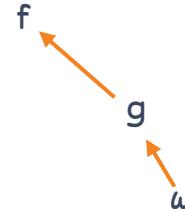
$$f \underline{o} g \quad \omega \iff f \underline{o} g \circ \omega$$



fög

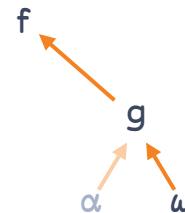


f ög f o g fög

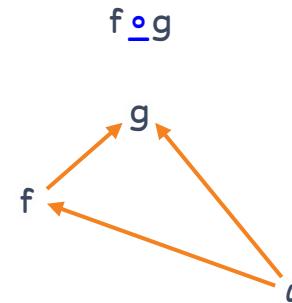


Function Composition

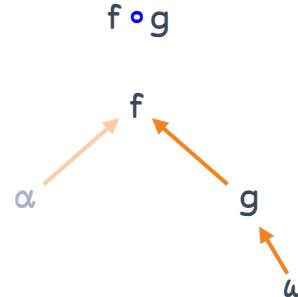
f ög



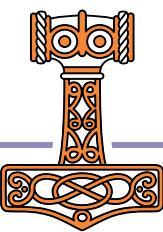
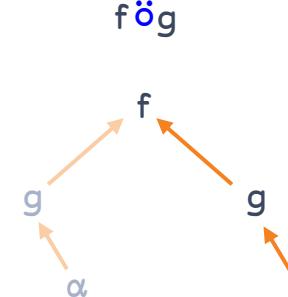
f o g



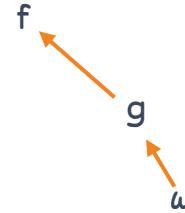
f o g



fög

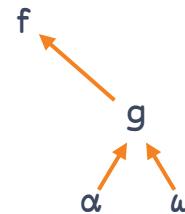


f ög f °g fög

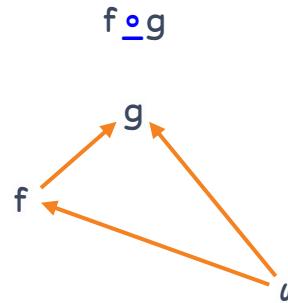


Function Composition

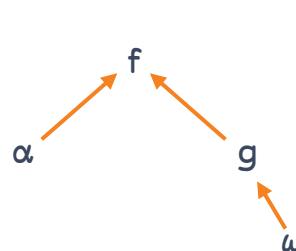
f ög



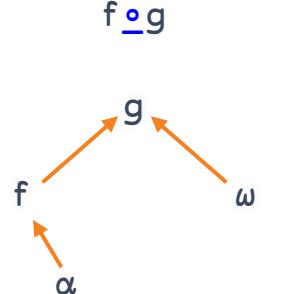
f °g



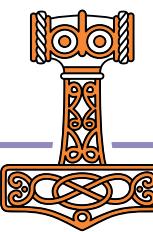
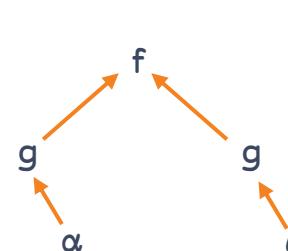
f °g



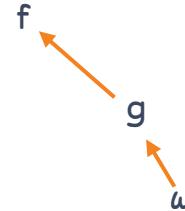
f °g



f ög

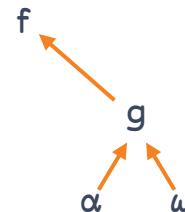


f ög f °g fög

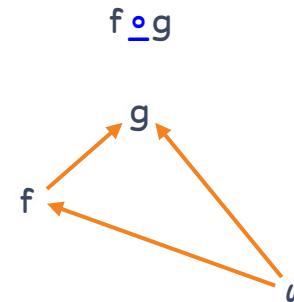


Function Composition

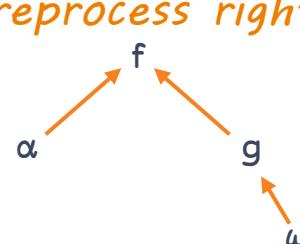
f ög



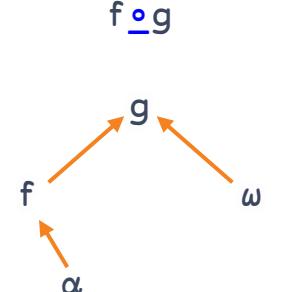
f °g



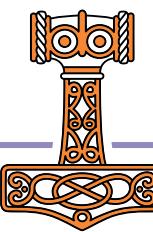
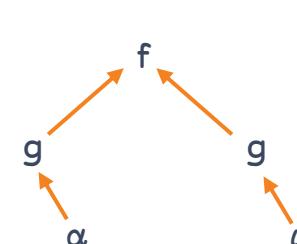
f °g
preprocess right



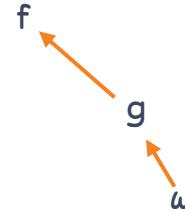
f °g



f ög

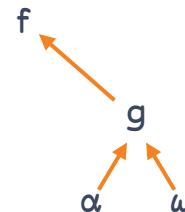


f ög f °g fög

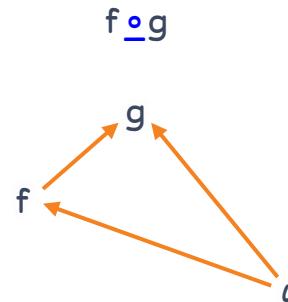


Function Composition

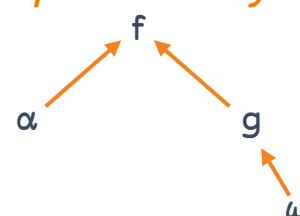
f ög



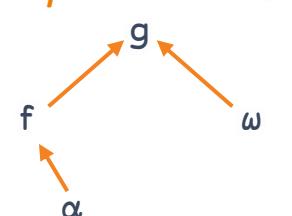
f °g



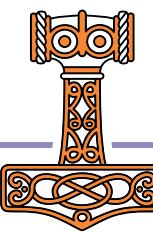
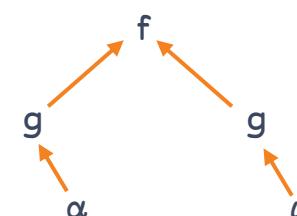
f °g
preprocess right



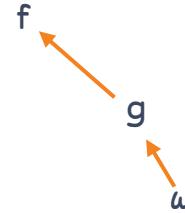
f °g
preprocess left



f ög

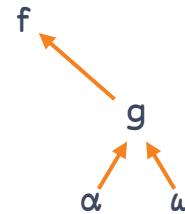


f ög f °g f ög

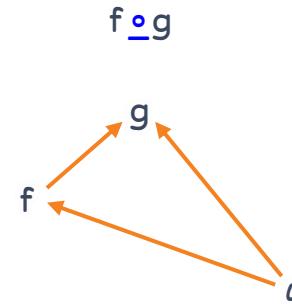


Function Composition

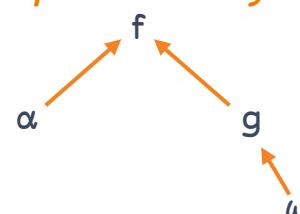
f ög



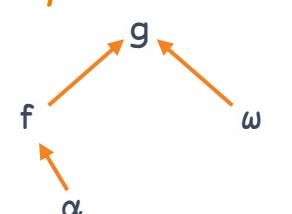
f °g



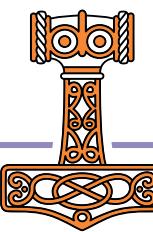
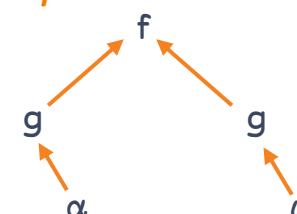
f °g
preprocess right



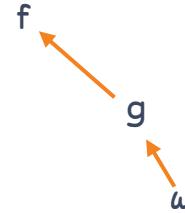
f °g
preprocess left



f ög
preprocess both

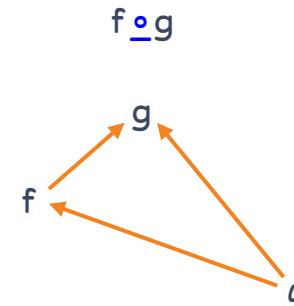


f ög f °g fög

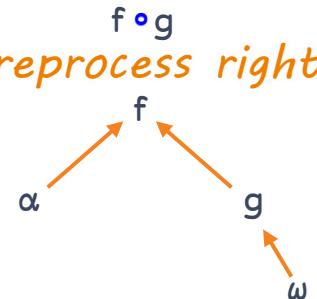


Function Composition

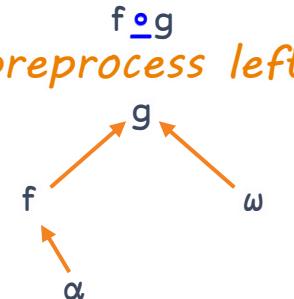
f ög
postprocess



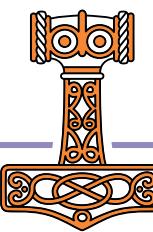
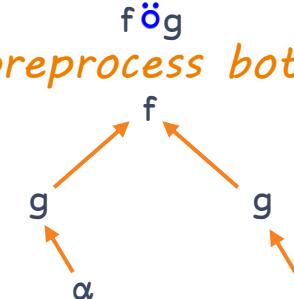
f °g
preprocess right



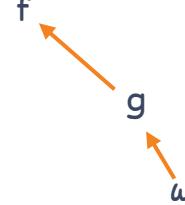
f °g
preprocess left



f ög
preprocess both

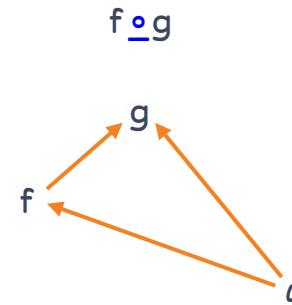


$f \circ g$ $f \circ g$ $f \circ g$
pre/postprocess

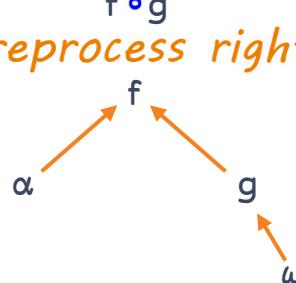


Function Composition

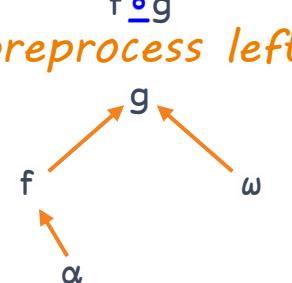
$f \circ g$
postprocess



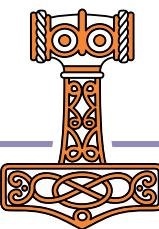
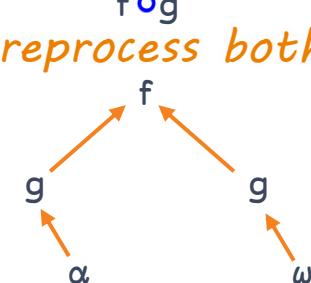
$f \circ g$
preprocess right



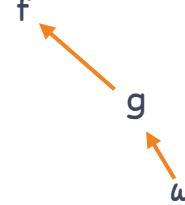
$f \circ g$
preprocess left



$f \circ g$
preprocess both

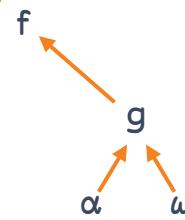


$f \circ g$ $f \circ g$ $f \circ g$
pre/postprocess

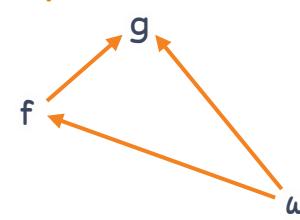


Function Composition

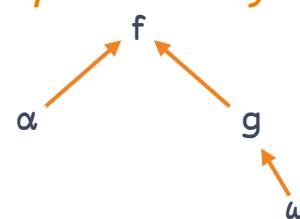
$f \circ g$
postprocess



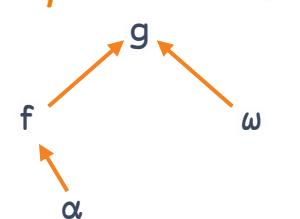
$f \circ g$
preprocess left



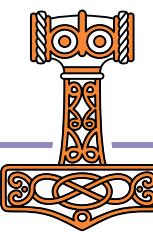
$f \circ g$
preprocess right



$f \circ g$
preprocess left



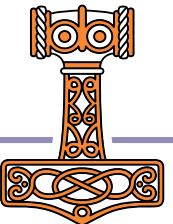
$f \circ g$
preprocess both



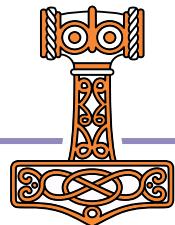
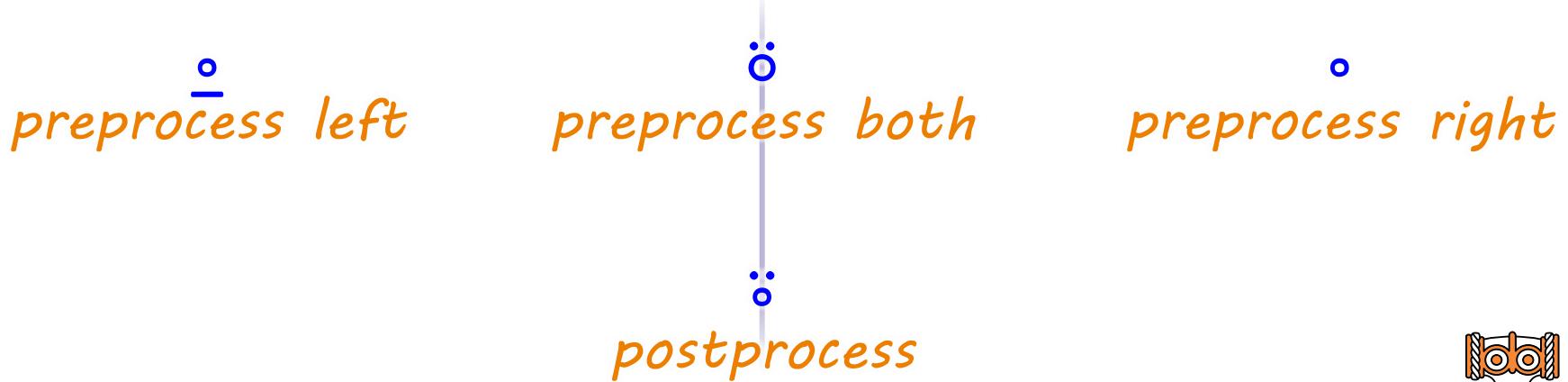
Function Composition

preprocess left *preprocess both* *preprocess right*

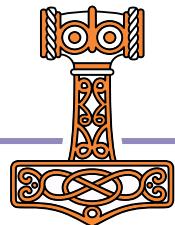
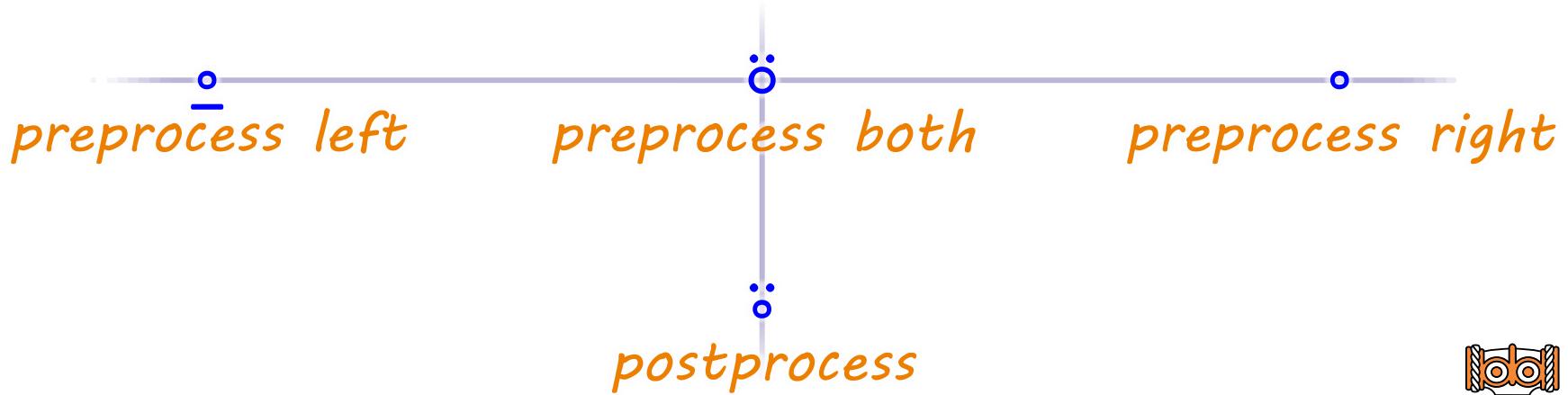
postprocess



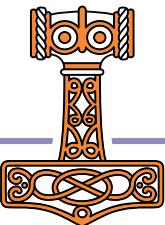
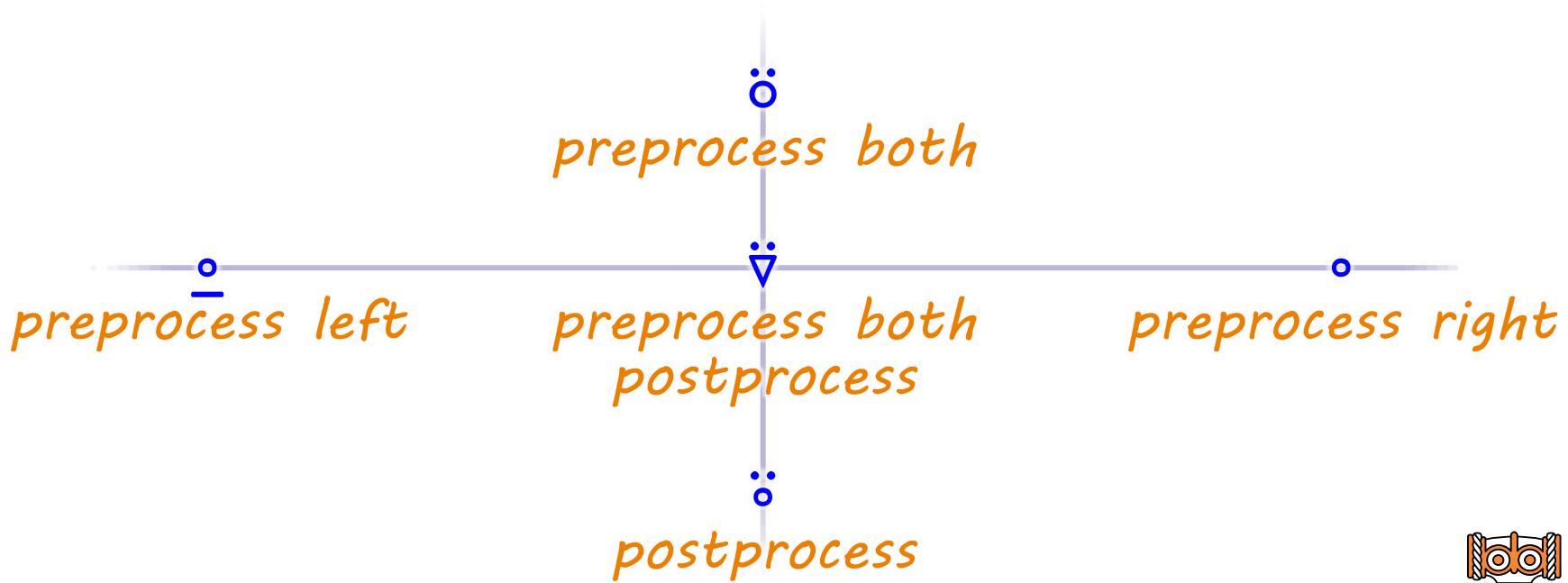
Function Composition



Function Composition

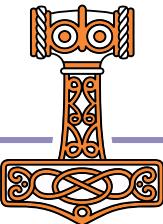


Function Composition



Function Composition

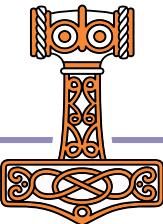
$f \circ g$ Behind with $X \sqsupseteq Y$ Select/Permute



Function Composition

$f \circ g$ Behind with $X \sqsupseteq Y$ Select/Permute

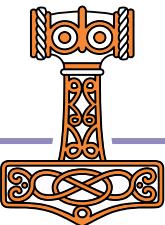
- ◆ Sort $\leftarrow (\Delta \sqsupseteq \vdash)$



Function Composition

$f \circ g$ Behind with $X \sqsupseteq Y$ Select/Permute

- ◆ Sort $\leftarrow \Delta \circ \Xi$

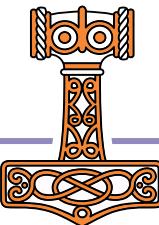


Function Composition

$f \circ g$ Behind with $X \sqsupseteq Y$ Select/Permute

Sort $\leftarrow \Delta \circ \Xi$

◆ Sorts $\leftarrow \Xi \circ \Delta$ ↗ "sort Y by X"

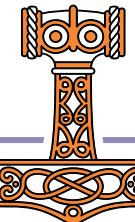


Function Composition

$f \circ g$ Behind with $X \sqsupseteq Y$ Select/Permute

Sort $\leftarrow \Delta^{\circ\Xi}$

◆ Sorts $\leftarrow \Delta^{\circ\Xi}$ a "sort Y by X"



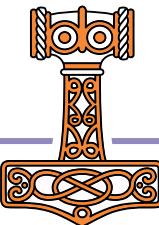
Function Composition

f \circ g Behind with $X \sqsupseteq Y$ Select/Permute

Sort $\leftarrow \Delta \circ \Xi$

Sorts $\leftarrow \Delta \circ \Xi$ ↗ "sort Y by X"

◆ Shuffle $\leftarrow (? \rightsquigarrow \circ \neq \sqsubseteq \vdash)$



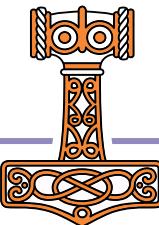
Function Composition

$f \circ g$ Behind with $X \sqsupseteq Y$ Select/Permute

Sort $\leftarrow \Delta^{\circ\Xi}$

Sorts $\leftarrow \Delta^{\circ\Xi}$ ↗ "sort Y by X"

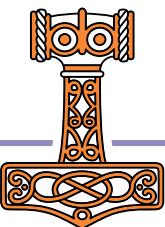
◆ Shuffle $\leftarrow ?\ddot{\circ} \circ \not{\Xi}^{\circ\Xi}$



Function Composition

$f \circ g$ Behind

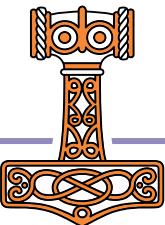
- ◆ SameAsFirst $\leftarrow \triangleright \circ =$
- ◆ HasDuplicates $\leftarrow \cup \circ \equiv$
- ◆ Palindrome $\leftarrow \phi \circ \equiv$
- ◆ IsPermutation $\leftarrow \triangleleft \circ \triangleleft \circ \equiv$



Function Composition

$f \circ g$ Behind

- ◆ Integer $\leftarrow L \circ =$
- ◆ Split $\leftarrow (\triangleright \circ \neq \subseteq \vdash)$
- ◆ Scale $\leftarrow \lceil / \circ (\div \ddot{\wedge})$
- ◆ Deviation $\leftarrow (+ \neq \div \neq) \circ (- \ddot{\wedge})$



Function Composition

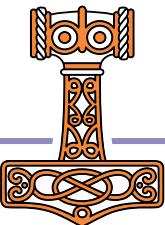
f o g Behind

Integer $\leftarrow \lfloor _ \rfloor =$

◆ Split $\leftarrow (\triangleright \underline{\circ} \neq \subseteq \vdash)$

Scale $\leftarrow \lceil / \underline{\circ} (\div \ddot{\cdot})$

Deviation $\leftarrow (+ \not\div \neq) \underline{\circ} (- \ddot{\cdot})$



Function Composition

$f \circ g$ Behind

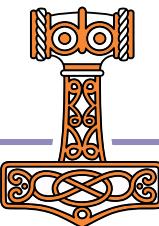
Integer $\leftarrow \lfloor \underline{\circ} =$

◆ Split $\leftarrow (\triangleright \underline{\circ} \neq \leq \vdash)$

Scale $\leftarrow \lceil / \underline{\circ} (\div \ddot{\wedge})$

Deviation $\leftarrow (+ \not\div \neq) \underline{\circ} (- \ddot{\wedge})$

' / ' ($\neq \leq \vdash$) 'hi/how/goes'
($\triangleright \underline{\circ} \neq \leq \vdash$) '/hi/how/goes'
' / ' ($\triangleright \underline{\circ} \neq \leq \vdash$) '/hi/how/goes'



Function Composition

$f \circ g$ Behind

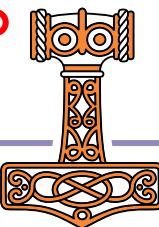
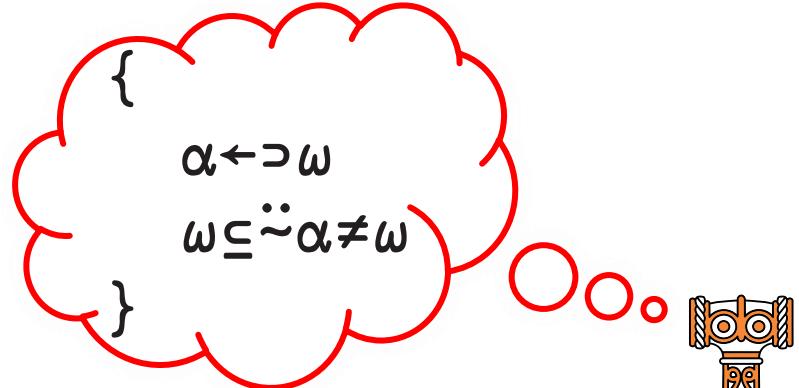
Integer $\leftarrow \lfloor \underline{\circ} =$

Split $\leftarrow (\triangleright \underline{\circ} \neq \leq \vdash)$

Scale $\leftarrow \lceil / \underline{\circ} (\div \ddot{\wedge})$

Deviation $\leftarrow (+/\div \neq) \underline{\circ} (-\ddot{\wedge})$

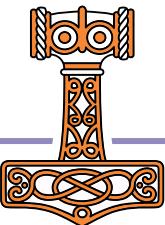
' / ' ($\neq \leq \vdash$) 'hi/how/goes'
($\triangleright \underline{\circ} \neq \leq \vdash$) '/hi/how/goes'
' / ' ($\triangleright \underline{\circ} \neq \leq \vdash$) '/hi/how/goes'



Function Composition

f o g Behind

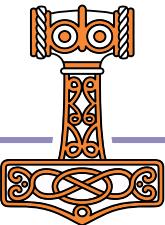
- ◆ Integer $\leftarrow \lfloor _ \rfloor =$
- ◆ Split $\leftarrow (\triangleright \underline{\circ} \neq \subseteq \vdash)$
- ◆ Scale $\leftarrow \lceil / \underline{\circ} (\div \ddot{\wedge})$
- ◆ Deviation $\leftarrow (+ \not\div \neq) \underline{\circ} (- \ddot{\wedge})$



Function Composition

f o g Behind

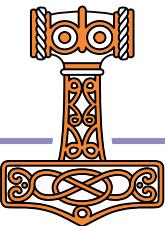
Filters \leftarrow o f



Function Composition

f o g Behind

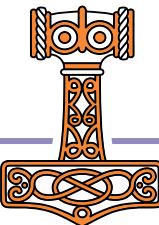
```
Filters ← of  
> o5 Filters 2 7 1 8 2 8  
7 8 8
```



Function Composition

f o g Behind

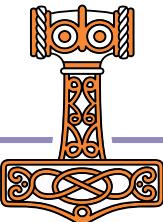
```
Filters ← o f
> o 5 Filters 2 7 1 8 2 8
7 8 8
    φo≡ Filters 'racecar' 'racer' 'toot'
racecar   toot
```



Function Composition

f o g Behind

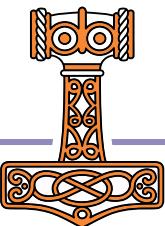
```
> o5 o/ 2 7 1 8 2 8  
7 8 8  
phio == o/ 'racecar' 'racer' 'toot'  
racecar    toot
```



Function Composition

$f \circ g$ Behind

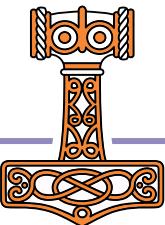
- ◆ Whence $\leftarrow \iota \circ \epsilon$ $\in \{(\iota\alpha) \in \omega\}$
- ◆ InPoly $\leftarrow \neg \circ \perp$ $\in \{(\neg\alpha) \perp \omega\}$
- ◆ Shapes $\leftarrow \rho \circ \rho$ $\in \{(\rho\alpha) \rho \omega\}$
- ◆ ToFile $\leftarrow \lhd \circ \square \text{NP}UT$ $\in \{(\lhd\alpha) \square \text{NP}UT \omega\}$



Function Composition

$f \circ g$ Behind

- ◆ FCat $\leftarrow \Phi \circ ,$
- ◆ RIndex $\leftarrow \Theta \circ \iota \quad \text{A } \{(\Theta\alpha) \iota \omega\}$
- ◆ RDrop $\leftarrow - \circ \downarrow \quad \text{A } \{(-\alpha) \downarrow \omega\}$
- ◆ RndSfx $\leftarrow - \ddot{\circ} ? \circ \uparrow \quad \text{A } \{(-?\alpha) \uparrow \omega\}$



Function Composition

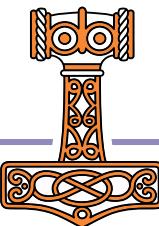
$f \circ g$ Behind

◆ FCat $\leftarrow \Phi \circ ,$

RIndex $\leftarrow \Theta \circ \iota$ $\in \{(\Theta\alpha)\iota\omega\}$

RDrop $\leftarrow -\circ \downarrow$ $\in \{(-\alpha)\downarrow\omega\}$

RndSfx $\leftarrow -\ddot{\circ} ? \circ \uparrow$ $\in \{(-?\alpha)\uparrow\omega\}$



Function Composition

$f \circ g$ Behind

FCat $\leftarrow \Phi \underline{\circ},$

RIndex $\leftarrow \Theta \underline{\circ} \iota$

RDrop $\leftarrow -\underline{\circ} \downarrow$

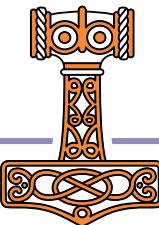
RndSfx $\leftarrow -\ddot{\circ} ? \underline{\circ} \uparrow$

APL\360: 'abc';42;'def'
Dyalog: 'abc',(42),'def'
19.0?: 'abc',42 $\Phi \underline{\circ}$, 'def'

A $\{(\Theta \alpha) \iota \omega\}$

A $\{(-\alpha) \downarrow \omega\}$

A $\{(-?\alpha) \uparrow \omega\}$

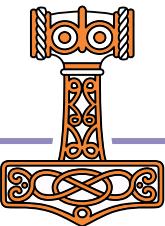


Function Composition

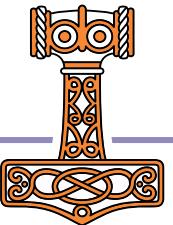
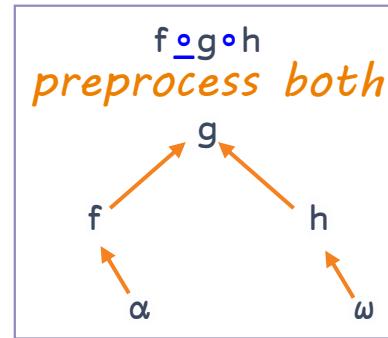
$f \circ g$ Behind

Split-compose

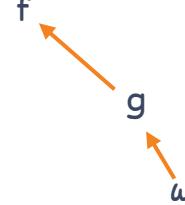
X $f \circ g \circ h$ Y



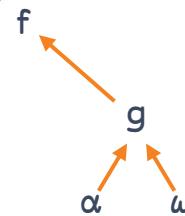
Function Composition



$f \circ g$ $f \circ g$ $f \circ g$
pre/postprocess

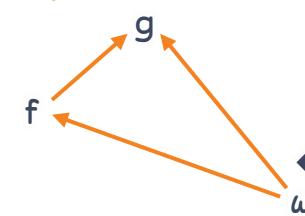


$f \circ g$
postprocess

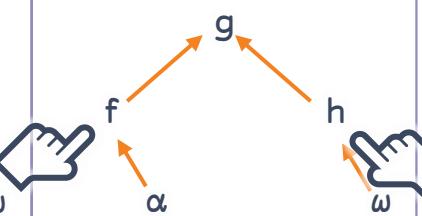


Function Composition

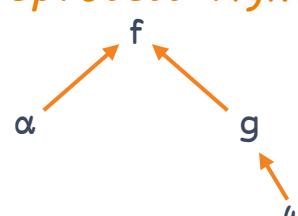
$f \circ g$
preprocess left



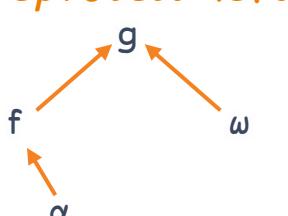
$f \circ g \circ h$
preprocess both



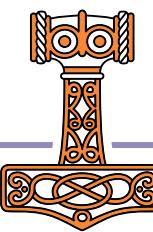
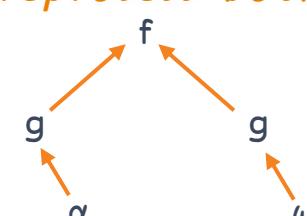
$f \circ g$
preprocess right



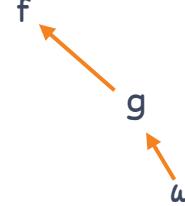
$f \circ g$
preprocess left



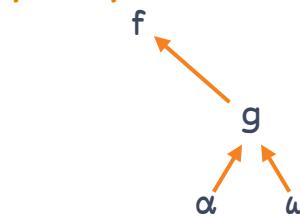
$f \circ g$
preprocess both



f ög f og fög
pre/postprocess

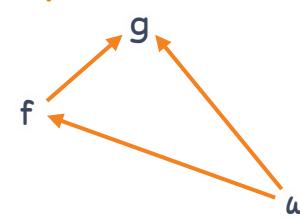


f ög
postprocess

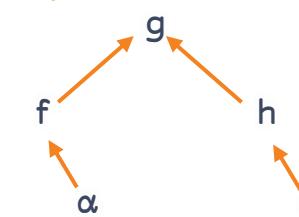


Function Composition

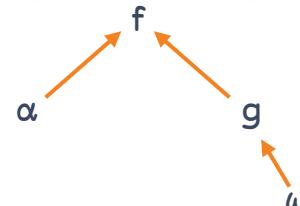
f og
preprocess left



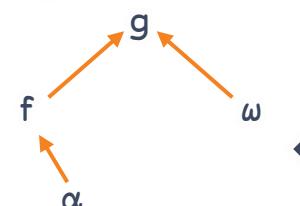
f og oh
preprocess both



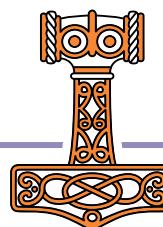
f og
preprocess right



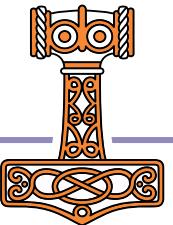
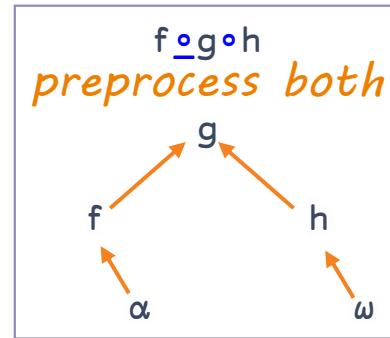
f og
preprocess left



$$\alpha \text{ } g \circ f \circ g \text{ } \omega \Leftrightarrow \alpha \text{ } f \circ g \text{ } \omega$$



Function Composition

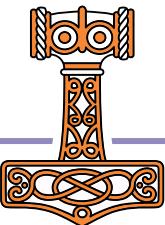


Function Composition

$f \circ g$ Behind

Split-compose

X $f \circ g \circ h$ Y



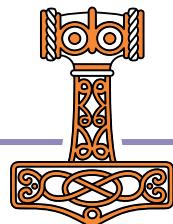
Function Composition

$f \circ g$ Behind

Split-compose

X $f \circ g \circ h$ Y

Pre-18.0: $g \circ f \circ h$
18.0: $(f \circ g \circ h)$
19.0?: $f \circ g \circ h$



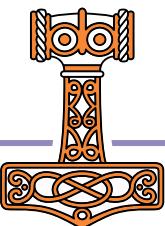
Function Composition

$f \circ g$ Behind

Split-compose

X $f \circ g \circ h$ Y

Pre-18.0: $((f \dashv)g(h \vdash))$
18.0: $(f \ddot{\circ} \dashv\ g\ h \ddot{\circ} \vdash)$
19.0?: $f \circ g \circ h$

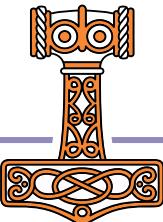


Function Composition

$f \circ g$ Behind

Split-compose $X \ f \circ g \circ h \ Y$

Hybrid mitigation $2 \circ | \circ /$



Function Composition

$f \circ g$ Behind

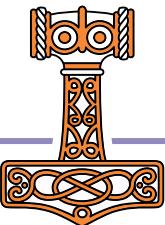
Split-compose

X $f \circ g \circ h$ Y

Hybrid mitigation

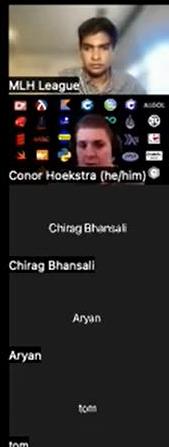
$2 \circ | \circ /$

Pre-18.0: $(2 \circ | \{\alpha/\omega\} \vdash)$
18.0: $(2 \circ | \vdash \circ / \vdash)$
19.0?: $2 \circ | \circ /$





BQN, which is my
favourite language 

$$\{((2|\omega)/\omega)\}$$
$$\{\omega/\tilde{\sim}2|\omega\}$$
$$(2|\vdash)\vdash\ddot{o}/\vdash$$
$$\{((2|x)/x)\}$$
$$\{x/\sim 2|x\}$$
$$(2|\vdash)/\vdash$$




BQN, which is my
favourite language

$$\{((2|\omega)/\omega)\}$$
$$\{\omega/\tilde{\sim}2|\omega\}$$
$$(2|\vdash)\vdash\ddot{o}/\vdash$$
$$\{((2|x)/x)\}$$
$$\{x/\tilde{\sim}2|x\}$$
$$(2|\vdash)/\vdash$$
$$\vdash/\tilde{\sim}2|\vdash$$

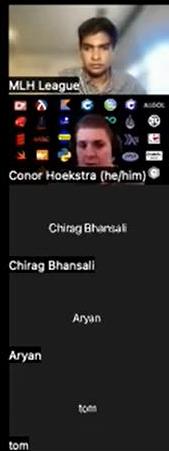



BQN, which is my
favourite language

$$\{((2|\omega)/\omega)\}$$
$$\{\omega/\tilde{\sim}2|\omega\}$$
$$(2|\vdash)\vdash\ddot{o}/\vdash$$
$$\{((2|\mathbb{X})/\mathbb{X})\}$$
$$\{\mathbb{X}/\tilde{\sim}2|\mathbb{X}\}$$
$$(2|\vdash)/\vdash$$
$$(2|\vdash)\circ/$$




BQN, which is my
favourite language

$$\{((2|\omega)/\omega)\}$$
$$\{\omega/\tilde{\sim}2|\omega\}$$
$$(2|\vdash)\vdash\ddot{o}/\vdash$$
$$\{((2|\mathbb{X})/\mathbb{X})\}$$
$$\{\mathbb{X}/\sim2|\mathbb{X}\}$$
$$(2|\vdash)/\vdash$$
$$(2|\vdash)\multimap/$$
$$2\multimap|\multimap/$$




BQN, which is my
favourite language

$$\{((2|\omega)/\omega)\}$$
$$\{\omega/\tilde{\sim}2|\omega\}$$
$$(2|\vdash)\vdash\ddot{o}/\vdash$$
$$(2|\vdash)\underline{o}/$$
$$\{((2|\mathbb{X})/\mathbb{X})\}$$
$$\{\mathbb{X}/\tilde{\sim}2|\mathbb{X}\}$$
$$(2|\vdash)/\vdash$$
$$(2|\vdash)\circ/\circ$$
$$2\circ|\circ/$$




BQN, which is my
favourite language 

$$\{((2|\omega)/\omega)\}$$
$$\{\omega/\tilde{\sim}2|\omega\}$$
$$(2|\vdash)\vdash\ddot{o}/\vdash$$
$$(2|\vdash)\underline{o}/$$
$$2\circ|\underline{o}/$$
$$\{((2|x)/x)\}$$
$$\{x/\tilde{\sim}2|x\}$$
$$(2|\vdash)/\vdash$$
$$(2|\vdash)\circ/$$
$$2\circ|\circ/$$


Function Composition

$f \circ g$ Behind

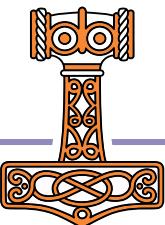
Split-compose

X $f \circ g \circ h$ Y

Hybrid mitigation

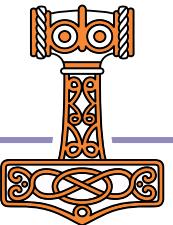
$2 \circ | \circ /$

Pre-18.0: $(2 \circ | \{\alpha/\omega\} \vdash)$
18.0: $(2 \circ | \vdash \circ / \vdash)$
19.0?: $2 \circ | \circ /$



Function Composition

$f \circ g$



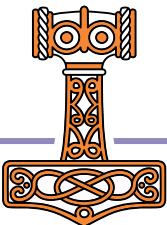
Core Language

Data Transformation

Function Application

Function Composition

$f \circ g$



Core Language

Data Transformation

$X \times Y$

ϕY

$X \sqcap Y$

$X \sqsupseteq Y$

Function Application

$f \not\in$

$f \ddot{*} g$

$f \ddot{o} k$

$f \ddot{e} k$

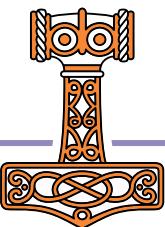
Function Composition

$f \ddot{o} g$

$f \ddot{o} g$

$f \circ g$

$f \underline{o} g$



Core Language

Data Transformation

Select

$Y[X;;]$

$X \sqsupseteq Y$

Function Application

Depth

$X f \cdots \lll Y$

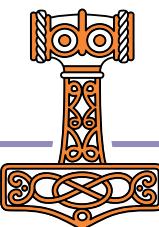
$X f \ddot{o} k Y$

Function Composition

Behind

$(f X)g Y$

$X f \underline{o} g Y$



Core Language

DRAFT PROPOSAL

Data Transformation

Select

$Y[X;;]$

$X \sqsupseteq Y$

<apl.wiki/select>

Function Application

Depth

$X f \cdots \ll Y$

$X f \ddot{o} k Y$

apl.wiki/depth_operator

Want it?

Function Composition

Behind

$(f X) g Y$

$X f \circ g Y$

<apl.wiki/behind>

Questions?

